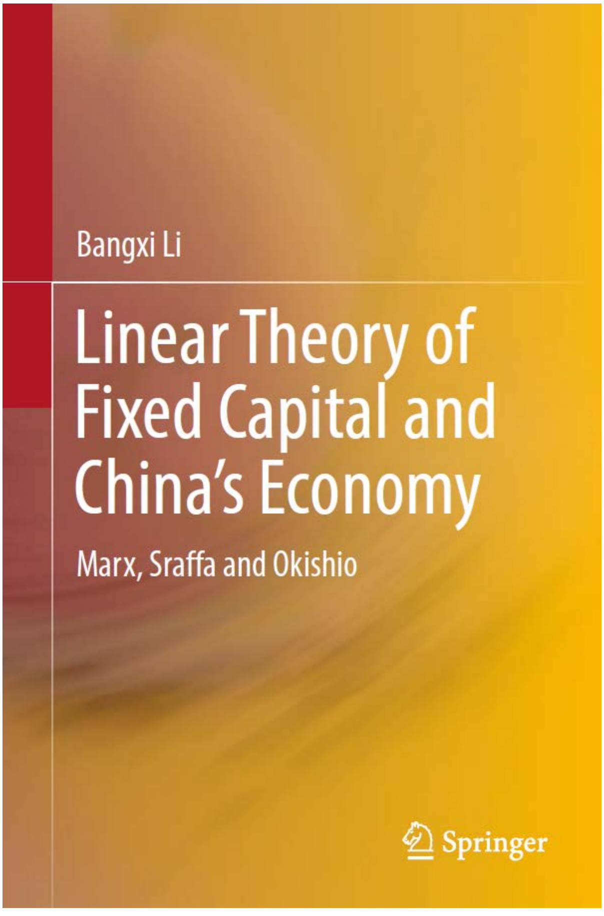
Bangxi Li

# Linear Theory of Fixed Capital and China's Economy

Marx, Sraffa and Okishio

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# Preface

This book is a collection of papers that comprises the doctorate dissertation of the author defended at Waseda University, June 2012. In addition, some papers published after the defence are included. Those are listed in the Bibliography part at the end of this book.

The author tried to do his best in arranging the contents of the papers in a readable way. As a result, the book consists of 10 chapters. Most of chapters were published as independent articles, so that they contained a lot of duplicated descriptions. The author tried to minimise duplications throughout the book, but some still remain for the convenience of readers.

A brief and overall introduction of the volume will be given in what follows.

The following chapters are all related to fixed capital theory and China's economy.

The first part of the book deals with the theory of fixed capital along the line of Marx, Sraffa, Okishio and Morishima. The second half is dedicated to estimation of fixed capital coefficients of China's economy and some computation results on China based on the estimated coefficients. Details will be given in Chap. 1, Introduction.

Most of theories of fixed capital so far discussed are more or less non-operational ones because there was no available data of fixed capital coefficients. Applications of Marx's labour theory of value have been made with no-joint-production models that ignore fixed capital. Models dealt with in this book are all linear multi-sector models, with a focus on fixed capital. Some details of fixed capital are made clear in a systematic approach, and this leads us to carry out several computations and simulations with explicit consideration of fixed capital.

The appendix is included in this book, because it reveals the motivation and the starting point of author's research in this field.

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Beijing, China Bangxi Li

# Chapter 1
# Introduction

This chapter gives the overall description of the following chapters. We also describe the system of notations employed in this volume.

## 1.1 Brief History of Mathematical Marx's Political Economy in Japan

If we look back some historical background of Mathematical Marx's political economy (MMPE) in Japan, from which the author gets his motivation of papers contained in this volume, we find that many ample intellectual assets have been acquired.

MMPE has been deeply discussed in Japan over several decades. Japan has been, indeed, one of the major centres of research in Marx's political economy. Hence, we focus on Japan. One can mention some names of Japanese researchers who contributed mathematical development of Marx's economics, such as Kei Shibata, Nobuo Okishio and Michio Morishima. Most of the fundamental results were established by N. Okishio. Okishio published a series of articles and volumes on Marx's political economy. In this sense Okishio is the pioneer of MMPE. MMPE established by Okishio has been further developed and deepend by his next generation, say Fujimori (1982), Nakatani (1994) and others.

Okishio initiated systematic mathematical analysis of Marx's political economy. He gave basic solutions to most of major problems within the framework of the so-called Leontief model. In addition to the above, by studying Keynes' economics, Okishio made remarkable contributions to instability and disequilibrium of capitalist economies. His theory of accumulation of capital and instability of capitalist economies, however, will not be dealt with in this volume.

Morishima (1973) reformed Okishio's theory of MMPE in the framework of dual dualities, *i.e.*, the systems of values, production prices and equilibrium growth, with the joint production setting. In Morishima's writings, notations and symbols, and



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ways of expressing systems of equations were greatly brushed up, and thus MMPE became very accessible for scientists and engineers.

It is not too much to say that fundamental points on linear economic models *à la* Leontief have been approaching to its apex. It is inevitable to generalise the framework of linear theory of political economy. It is not difficult to find images of joint production in the real economy, and the joint production system itself was dealt with by von Neumann (1945/46[1937]) earlier. If we shift our focus from no joint production systems *à la* Leontief to joint production systems, however, various problems become much harder, because the framework of analysis is usually based on rectangular coefficient matrices, and hence formulation is often made in terms of the system of inequalities. Then, the first major mathematical tools are linear programming, topology and so forth.

Morishima (1973) cast three problems on MMPE with respect to the so-called FMT, namely, joint production, fixed capital and reduction of skilled labour to unskilled labour. It seems that Fujimori (1982) is the first to present answers to Morishima's problems in the dual dualities framework.

With respect to positive analysis of Japan's economy in the light of linear theory of Marx's political economy, Okishio and Okishio-Nakatani challenged the computation of labour values of commodities produced in Japan. Their analysis is, however, limited to the Leontief system.

Okishio-Nakatanai (1975), Fujimori (1982) and Nakatani (1994) treated problems of fixed capital in MMPE, but some problems seems to be left unsolved. Nonetheless, the basis for computing labour values and production prices of commodities in the system with fixed capital was ignited in these period. This collection of papers starts from the theory of fixed capital.

Our main concern is the system with fixed capital of various ages, and we will see some positive analysis of China's economy with fixed capital.

It seems that D. Ricardo started some systematic discussion on machines. K. Marx treated the simple reproduction of fixed capital.

After that, fixed capital had long been treated with the framework of depreciation as a cost, and its computation is usually very complicated in many literatures. It is often assumed that fixed capital holds the identical efficiency while it is operable. Differences among fixed capital is evaluated in the form of depreciation behind the production process. Although fixed capital shares the same function in the process of production, however, if the working age is different, it is possible to make a distinction among fixed capital based on the different ages. Fixed capital of age 0, *i.e.*, newborn fixed capital, becomes the fixed capital of age 1 after one year of operation. In other words, when fixed capital is employed for producing commodities in the production process, while main products of some sort are manufactured as outputs, fixed capital that is one year older than the employed one is also produced. That is to say, this production process produces two or more types of commodities, and is a joint production process. Therefore, we can analyze and examine the problem of fixed capital with the framework of joint production.

Sraffa (1960) seems to be the first scholar that treats fixed capital in the joint production framework with equalities. The system of simultaneous equations has rectangular coefficient matrices, so that the search for the equilibrium profit rate and the equilibrium production price system needs some technical tricks.

Equilibrium of prices with the uniform rate of profit comprises all the commodities including aged fixed capital. Sraffa (1960) then showed that equilibrium prices of aged fixed capital can be eliminated from the system, and one obtains the equilibrium system of production prices of commodities without aged fixed capital, which might be called reduction of the whole commodity system its subsystem with brand-new commodities alone. Okishio and Nakatani (1975) discussed this reduction for Marx's production price system.

After Sraffa (1960), Okishio and Nakatani (1975) showed that the system of simultaneous equations to define the profit rate and the equilibrium price of all types of commodities can be reduced to the subsystem of brand-new commodities, but the resulting eigensystem is not operational for examining stability of equilibrium. Many a literature discussed this reduction. Refer to *e.g.* Fujimori (1982), Schefold (1989), Kurz and Salvadori (1995), *etc*.

## 1.2 Plan of Chapters

This book first investigate the problem of reproduction including fixed capital. The major points of this small book are based on the theory of fixed capital. Hence, the first part of the book is dedicated to development of theory of fixed capital. In the second part, we apply the theory to positive analysis of China's economy.

**First part** The objective of Chap. 2 is to make clear some mathematical and/or formal features of their reduction procedures.

The equilibrium production price and equilibrium quantity system framework including fixed capital, which will be introduced below, have the prerequisite of pre-paid wages; from this perspective, it is within the Marxian paradigm. On the other hand, analyzing and examining the problem of fixed capital from the perspective of joint production is within the Sraffian paradigm. Therefore, we named the model as a narrowly defined Marx-Sraffa model.

Based upon Sraffa's idea, Okishio and Nakatani (1975) developed this narrowly defined Marx-Sraffa model; specifically, they simplified the joint production system, which include both brandnew and aged fixed capital as a subsystem with only new commodities. We shall label this subsystem as the Sraffa-Okishio-Nakatani economy, in short SON (Asada 1982; Li 2012).

Chapter 2 will discuss the formality of the reduction of the whole economy with aged fixed capital to its subsystem of brandnew commodity world.

In the outset, we deal with the formality of this reduction from the angle of non-singular transformation of the matrix concerned. The rectangular coefficient matrix of the price system as a matrix pencil will be transformed into a matrix with three blocks. One of the blocks concerns the subworld of brandnew commodities. The next block is related to the determination of prices of aged fixed capital, namely depreciation. The third block is the zero block.

In Chap. 3, we focus on depreciation and renewal dynamics of fixed capital. Since the well-known exchange of views between Marx and Engels, many arguments have been made as to the role of depreciation of fixed capital and its reinvestment. Some deal with depreciation as costs, others do as a source of investment.

It will be shown that renewal dynamics is a sort of the Markov process. One important feature of the process is that the dominant eigenvalue is duplicated. We will see that renewal dynamics of fixed capital is converging, but very slowly. Even if accelerated depreciation is made by firms, the basic characteristics of dynamics need not to be altered. We employ simulation methods in this chapter.

Chapter 4 investigates the dual system of profits and growth in a very simplified system. By applying the reduction of the whole commodity world to its subworld without aged fixed capital both in the production price and the quantity systems, one can extend the so-called Cambridge equation by Pasinetti.

In fact, the rate of profit and the rate of accumulation defined by net profit do not satisfy the Cambridge equation (Fujimori-Li 2010) under this subsystem.

The system here consists of only one basic type of fixed capital and reproduces itself. We start from the whole commodity world including aged fixed capital. Aged fixed capital matters to both profit and growth. We apply the same reduction to the system of quantities. It seems that Fujimori (1982) alone paid attention to the quantity side of the above reduction procedure.

We combine the dual system of profits and growth, and to reestablish the Cambridge equation by Pasinetti. The profit rate measured in the subworld, SON, comprises depreciation of fixed capital, namely profits emerge as internal reserves.

It will be shown that the Cambridge equation can be rebuilt in the same formality, if we define the concept of gross profit by internal reserve, that contains the amount of depreciation, and define the rate of accumulation by gross investment divided by gross profit, accordingly.

In Chap. 5, we discuss the decreasing efficiency of fixed capital with aging and its durability by means of simulation. It will be shown that durability of fixed capital is determined by the rate of profit. We simulate the hardening effect of fixed capital.

In Chap. 6, one will see that the equilibria of the dual system of profits and growth of commodity production are found as the eigenvalue and corresponding eigen vectors of the system obtained by applying the Moore-Penrose psudo-inverse. Even if the analysis starts from the system of inequalities, the equilibrium point can be defined by the system of equalities. Hence, we may put our main focus of research on the system of equalities.

Existence of equilibrium can be dealt with by linear programming, but stability is not well discussed by that. Our new approach in this chapter will present a direct method to evaluate eigenvalues and corresponding eigenvectors of the system of all commodities at once, thus gives an operational analysis of the Marx-Sraffa model. The resulting eigensystems shows that both equilibria will be possibly unstable.

Dual (in)stability (Jorgenson 1960) of growth and profit is one of important theoretical results in linear economic models dealt with in this volume. That is, the equilibrium production prices are stable, while equilibrium of quantities that needs to clear demand and supply may be unstable. Hua (1984) discussed the dual instability from the angle of the planned economy. The central point of dual instability is that the framework of analysis is focused on the eigenvalues and eigenvectors of square, nonnegative coefficient matrices of the system concerned.

**Second part** After theory of fixed capital, we begin with its application to China's actual economy. The first thing to be done is the preparation of data on fixed capital.

Chapter 7 deals with the estimation of marginal fixed capital coefficients from the gross investment matrices of China's economy 1995–2000. The method developed by Fujimori (1992b) is slightly amended, and applied to data of China's economy. In the process of estimating marginal fixed capital coefficients, we draw the wage-profit curves of China's economy, and we try to locate the position of China's economy for the years concerned.

Chapter 8 treats the computation of labour values of commodities in China. In lieu of the organic composition of capital, we focus on the fixed-capital/labour ratio. Since it is not easy to establish generalised results on the relationship between production-price/labour value ratios and the fixed-capital/labour ratios, we compute Speerman's coefficients of the two.

We make clear one possible implication of Marx's transformation proposition applied to the agriculture of China's economy.

Chapter 9 discusses the turnpike path of China's economy. Usually, the turnpike path is simulated *ex ante* for future planning. The simulation that is carried out here is, however, a computation for the past, the one *ex post*. This implies the evaluation of the past economic policy guided by the Chinese government. It will be shown that the actural path of growth of China's economy of the years concerned is close to the turnpike of the period, with some evidence of overheating.

## 1.3 Basic Framework of the Marx-Sraffa Model

As said in the above, the book discusses the joint production system of aged fixed capital.

Joint-production systems have been dealt with linear inequalities for the sake of applying linear programming. In what follows, we mostly focus on the equality based linear economic models of Marx, Sraffa and Okishio. Hence, we describe here the fundamental framework of the models analysed in the following chapters.

Suppose that an economy consists of $m$ commodities and $n$ processes. Let $A$ and $B$ denote the input and the output matrices, respectively. $A$ and $B$ should be of the same dimension, $m \times n$.

Let $\ell$ and $f$ denote the labour vector $(1 \times n)$ and the wage goods bundle $(m \times 1)$, respectively. The augmented input matrix is then defined by $M = A + f\ell$. All those are assumed to be real.

Let $p$ represent $1 \times m$ price vector, and $\lambda$ the profit factor, and the price equilibrium *à la* Marx-Sraffa is defined by a pair of nontrivial $p \neq \mathbf{0}$ and $\lambda$ fulfilling the following:

$$pB = \lambda pM. \tag{1.3.1}$$

The problem may be regarded as an algebraic equation with $p$ and $\lambda$ as its unknown variables. As will be shown in the following, there are two ways to tackle this problem: *one* is to reduce the number of variables and equations, so that the system becomes simplified; *another* is to construct the eigensystem from $B - \lambda M$ directly, in view of equivalence between (1.3.1) and

$$p(B - \lambda M) = \mathbf{0}, \tag{1.3.2}$$

where $\mathbf{0}$ denotes a row zero vector of a relevant dimension.

Generally speaking, it is not easy to solve this type of the simultaneous system of equations with a rectangular coefficient matrix. A familiar model is therefore expressed in terms of inequalities in lieu of equalities.

One important remark is that the existence of equilibrium will not be discussed in this book. The book rather concentrates on how to search for equilibrium. It is assumed that there is an equilibrium of the system concerned.

In order to find solutions of (1.3.1), various additional settings and conditions should be made. The description about them is one of major points in the following sections.

Remark that most of symbols are defined and used locally in a narrow scope of description, if not otherwise stated. In representing the transposed matrices, we denote ${}^{t}X$, the transpose of $X$.

In almost all discussions made in this book, we *assume* that wages are paid exante, in accordance with Marx's idea. It is assumed that labour is homogeneous.

Our focus will be put on the equality system of equations related to equilibrium. In some cases, however, we employ the system of inequalities *à la* von Neumann.

# Chapter 2
# Sraffa-Okishio-Nakatani's Fixed Capital Theory

## 2.1 Introduction

It is known that although fixed capital shares the same function in the process of production, if the working age (or age) is different, one should make a distinction based on the different ages. The fixed capital at age 0 becomes the fixed capital at age 1 after 1 year of use. In other words, when fixed capital is used for commodities production in the production process, while products are manufactured as output, fixed capital that is 1 year older than the inputted fixed capital is also produced. That is to say, this production process produces two or more types of commodities and is a joint production process. Therefore, we can analyze and examine the problem of fixed capital by putting it in the joint production system.

Although the idea of joint-production is not nobel at all, Sraffa (1960) seems to be the first literature that treated fixed capital from the angle of joint production; fixed capital is distinguished by ages, namely, regarded as different commodities, and hence, aged fixed capital is grasped as jointly produced with the main commodities of the process that employs fixed capital. Okishio and Nakatani (1975) simplified this narrowly defined Marx-Sraffa model; specifically, they simplified the joint production system, which include both newborn and aged fixed capital as a subsystem with only brand-new commodities. We could label this subsystem as the Sraffa-Okishio-Nakatani economy, in short SON (Asada 1982; Li 2012).

## 2.2 Reduction *à la* Sraffa-Okishio-Nakatani

In the outset, Sraffa-Okishio-Nakatani's framework will be made clear by way of listing presuppositions.

**SON1** Only aged fixed capital is jointly produced by processes.
**SON2** There is no process that produces only aged fixed capital.



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**SON3** Irrespective of ages, technical efficiency of fixed capital remains the same while it is employed in production.

**SON4** Formation of processes obeys the generating rule of processes with fixed capital: a combination of fixed capital starts from the one with brand-new fixed capital alone; in the next period the process will be equipped with 1 year old fixed capital; if some of fixed capital reach their durability, they will be replaced by brand-new one.

The generating rule in SON4 excludes the possibility that worn-out fixed capital is replaced by aged fixed capital. This implies that there is no secondhand market of fixed capital. Besides, from this rule, the number of processes of a sector which produces one type of brand-new commodity does not exceed the minimal multiplier of durabilities of fixed capital in an economy. Intuitively, this implies the number of processes of a whole economy is larger than the number of commodities.

Additionally, *no-cost scrapping* and *exogenously given durability* are assumed with respect to fixed capital.

Sraffa established the prototype of reduction. He started from the system of simultaneous equations concerning all processes and all types of commodities, both brand-new ones and aged fixed capital, and then reduced the system to the one with brand-new commodities.

Okishio and Nakatani (1975) extended Sraffa's approach to the case of Marxian models. Okishio-Nakatani showed that prices of brand-new commodities and equilibrium profit rates are determined in the reduced system of brand-new commodities. Their approach seems to be the one to reduce the number of unknowns and the number of equations, and eventually reached to a system with square coefficient matrix.

In this section, Okishio-Nakatani's reduction procedure will be illustrated with simple examples, and some generalised viewpoint will be presented. Its relationship to replacement dynamics will be shown thereafter.

### *2.2.1 Okishio-Nakatani Reduction*

In the following, two simple examples of Okishio-Nakatani's reduction will be shown on the basis of (1.3.2).

This subsection presents a new aspect to the problem. A new point here is that the coefficient matrix $B - \lambda M$ will be decomposed from the angle of nonsingular transformation of matrices.

**One fixed capital case** Commodities other than fixed capital are disregarded. Suppose there is basically only one class of fixed capital. The durability is assumed to be 3 years. Fixed capitals are classified according to their ages; there are three types of fixed capital, namely, 0-year old, 1-year old and 2-year old fixed capital. In an economic system, it is assumed that the above introduced fixed capitals are reproduced. Formally, there is a system of production with 3 types of commodities.

They are indexed from 1 to 3. Thus, one obtains the following input and output matrices of the system:

$$M = \begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 & 1 \\ k & 0 & 0 \\ 0 & k & 0 \end{pmatrix},$$

and,

$$B - \lambda M = \begin{pmatrix} 1 - \lambda k & 1 & 1 \\ k & -\lambda k & 0 \\ 0 & k & -\lambda k \end{pmatrix}. \tag{2.2.1}$$

Form the diagonal matrix with $\lambda^2$, $\lambda$ and 1 as its diagonal elements, $\Lambda = \begin{pmatrix} \lambda^2 & & \\ & \lambda & \\ & & 1 \end{pmatrix}$ and postmultiply this to $B - \lambda M$, and one obtains:

$$(B - \lambda M)\Lambda = \begin{pmatrix} \lambda^2(1 - \lambda k) & \lambda & 1 \\ \lambda^2 k & -\lambda^2 k & 0 \\ 0 & \lambda k & -\lambda k \end{pmatrix}.$$

Then, postmultiply $E = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ to the above result, and it follows that

$$(B - \lambda M)\Lambda E \sim \begin{pmatrix} \zeta_{11}(\lambda) & \lambda + 1 & 1 \\ 0 & -\lambda^2 & 0 \\ 0 & 0 & -\lambda \end{pmatrix},$$

where $\zeta_{11}(\lambda) = -\lambda^3 k + \lambda^2 + \lambda + 1$.

In the above, if $\zeta_{11}(\lambda) = 0$, then $\operatorname{rank}(B - \lambda M)\Lambda E = \operatorname{rank}(B - \lambda M) = 2 < 3$, so that there exists a $p \neq \mathbf{0}$ such that $p(B - \lambda M) = \mathbf{0}$. The condition $\zeta_{11}(\lambda) = 0$ determines the profit factor as a function of technical coefficient $k$. The order of $\lambda$ is related to the durability of fixed capital.

Thus, the first column of $B - \lambda M$ determines the profit rate and non negative price of brandnew fixed capital. The remaining columns determines the relative ratio of prices of aged fixed capital to the price of brand-new fixed capital.

**Two types of fixed capital case** Since the above example is too simple, a more complicated case will be shown in the next place. Suppose there are two types of fixed capital, durabilities of them are 4 and 2 years, respectively. Commodities other than fixed capital are again ignored. When ages of fixed capital does not matter, two fixed capitals are identified; fixed capital 1 with durability 4, and fixed capital 2 with durability 2. We distinguish two sectors. The first sector produces fixed capital 1, while the second sector produces fixed capital 2: $B = (B_1, B_2)$ and $M = (M_1, M_2)$ with

$$M_j = \begin{pmatrix} k_{1j} & & & \\ & k_{1j} & & \\ & & k_{1j} & \\ & & & k_{1j} \\ k_{2j} & & k_{2j} & \\ & k_{2j} & & k_{2j} \end{pmatrix}, \quad B_j = \begin{pmatrix} \delta_{j1} & \delta_{j1} & \delta_{j1} & \delta_{j1} \\ k_{1j} & & & \\ & k_{1j} & & \\ & & k_{1j} & \\ \delta_{j2} & \delta_{j2} & \delta_{j2} & \delta_{j2} \\ k_{2j} & & k_{2j} & \end{pmatrix},$$

where $j = 1,\ 2$, and $\delta_{ij} = 1$ for $i = j$ or 0 otherwise.

For each $B_j - \lambda M_j$ block, one can apply the similar procedure: let

$$L = \begin{pmatrix} \lambda^3 & & & \\ & \lambda^2 & & \\ & & \lambda & \\ & & & 1 \end{pmatrix}, \quad E = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 1 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 1 & 1 & \dots & 1 \end{pmatrix},$$

and one can compute $(B_j - \lambda M_j)LE$. Then, for the sake of brevity, one should collect rows related to brandnew commodities in the top positions of the matrix. That is, the 4th row should be moved to the 2nd row position. This row-wise operation is expressed by pre-multiplication of an elementary matrix to $(B_j - \lambda M_j)LE$. Let $F$ denote such a matrix, and one obtains:

$$F(B_1 - \lambda M_1)LE = \begin{pmatrix} \psi_{11}(\lambda) & \lambda^2 + \lambda + 1 & \lambda + 1 & 1 \\ -k_{21}\lambda^2(\lambda^2 + 1) & -k_{21}\lambda^2 & -k_{21}\lambda^2 & 0 \\ 0 & -k_{11}\lambda^3 & 0 & 0 \\ 0 & 0 & -k_{11}\lambda^2 & 0 \\ 0 & 0 & 0 & -k_{11}\lambda \\ 0 & -k_{21}\lambda^3 & 0 & -k_{21}\lambda \end{pmatrix},$$

$$F(B_2 - \lambda M_2)LE = \begin{pmatrix} -k_{12}\lambda^4 & 0 & 0 & 0 \\ \psi_{25}(\lambda) & \psi_{26}(\lambda) & -k_{22}\lambda^2 + \lambda + 1 & 1 \\ 0 & -k_{12}\lambda^3 & 0 & 0 \\ 0 & 0 & -k_{12}\lambda^4 & 0 \\ 0 & 0 & 0 & -k_{12}\lambda \\ 0 & -k_{22}\lambda^3 & 0 & -k_{22}\lambda \end{pmatrix},$$

where $\psi_{ij}(\lambda)$s are polynomials of $\lambda$, and

$$\begin{aligned} \psi_{11}(\lambda) &= -k_{11}\lambda^6 + \lambda^3 + \lambda^2 + \lambda + 1, \\ \psi_{25}(\lambda) &= -k_{22}\lambda^4 + \lambda^3 + (1 - k_{22})\lambda^2 + \lambda + 1, \\ \psi_{26}(\lambda) &= (1 - k_{22})\lambda^2 + \lambda + 1. \end{aligned}$$

As an operation to the whole system, one should form diagonal matrices

$$\Lambda = \begin{pmatrix} L & \\ & L \end{pmatrix}, \quad \mathcal{E} = \begin{pmatrix} E & O \\ O & E \end{pmatrix},$$

and, one obtains

$$F(B - \lambda M)\Lambda\mathcal{E} = F\Big((B_1 - \lambda M_1)LE,\ (B_2 - \lambda M_2)LE\Big).$$

The final step is to collect all of the first column of each block to the leftmost side of the matrix. Let $\mathcal{G}$ denote the matrix that represents those columnwise interchanges, and one obtains $\mathcal{Z} = F(B - \lambda M)\Lambda\mathcal{E}\mathcal{G}$ as follows:

$$\mathcal{Z} = \left(\begin{array}{ccccc} \psi_{11}(\lambda) & -k_{12}\lambda^4 & \lambda^2+\lambda+1 & \lambda+1 & 1 \\ -k_{21}\lambda^2(\lambda^2+1) & \psi_{22}(\lambda) & -k_{21}\lambda^2 & -k_{21}\lambda^2 & 0 \\ 0 & 0 & -k_{11}\lambda^3 & 0 & 0 \\ 0 & 0 & 0 & -k_{11}\lambda^2 & 0 \\ 0 & 0 & 0 & 0 & -k_{11}\lambda \\ 0 & 0 & -k_{21}\lambda^3 & 0 & -k_{21}\lambda \end{array}\right.$$

$$\left.\begin{array}{ccc} 0 & 0 & 0 \\ \psi_{26}(\lambda) & -k_{22}\lambda^2+\lambda+1 & 1 \\ -k_{12}\lambda^3 & 0 & 0 \\ 0 & -k_{12}\lambda^4 & 0 \\ 0 & 0 & -k_{12}\lambda \\ -k_{22}\lambda^3 & 0 & -k_{22}\lambda \end{array}\right).$$

Since $F,\ \Lambda,\ \mathcal{E},\ \mathcal{G}$ are all nonsingular matrices, the above transformation is rank-preserving, so that $\operatorname{rank}(\mathcal{Z}) = \operatorname{rank}(B - \lambda M)$.

Take the leftmost two columns of $\mathcal{Z}$, and that block is related to only brand-new fixed capital. If $\lambda$ is such that $|\Psi(\lambda)| = \begin{vmatrix} \psi_{11}(\lambda) & -k_{12}\lambda^4 \\ -k_{21}\lambda^2(\lambda^2+1) & \psi_{25}(\lambda) \end{vmatrix} = 0$, then $\operatorname{rank}(\mathcal{Z}) < m$, and hence there exists a vector $p \neq \boldsymbol{\ominus}$ which fulfills $p(B - \lambda M) = \boldsymbol{\ominus}$ or $x \neq \mathbf{0}$ such that $(B - \lambda M)x = \mathbf{0}$. $\Psi(\lambda)$ is nothing but the coefficient matrix of reduced system of only brand-new commodities.

From the dimension of $\mathcal{Z}$, the number of linearly independent columns of $\mathcal{Z}$ never exceeds its number of rows, so that one has only to consider the matrix formed by column 1, 2, 3, 4, 5 and 6 of $\mathcal{Z}$, for instance.

$$\mathcal{Z}_1 = \begin{pmatrix} \psi_{11}(\lambda) & -k_{12}\lambda^4 & \lambda^2+\lambda+1 & \lambda+1 & 1 & 0 \\ -k_{21}\lambda^2(\lambda^2+1) & \psi_{25}(\lambda) & -k_{21}\lambda^2 & -k_{21}\lambda^2 & 0 & \psi_{26}(\lambda) \\ 0 & 0 & -k_{11}\lambda^3 & 0 & 0 & -k_{12}\lambda^3 \\ 0 & 0 & 0 & -k_{11}\lambda^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -k_{11}\lambda & 0 \\ 0 & 0 & -k_{21}\lambda^3 & 0 & -k_{21}\lambda & -k_{22}\lambda \end{pmatrix}.$$

By applying appropriate Gauss sweeping-out procedures, one obtains:

$$\mathcal{Z}_1 \sim \mathcal{Z}_1^* = \begin{pmatrix} \psi_{11}(\lambda) & -k_{12}\lambda^4 & \lambda^2+\lambda+1 & \lambda+1 & 1 & 0 \\ -k_{21}\lambda^2(\lambda^2+1) & \psi_{25}(\lambda) & 0 & 0 & 0 & \lambda \\ 0 & 0 & -\lambda^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\lambda^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & -\lambda^2 \end{pmatrix}.$$

Remark that in view of the dimension of the matrix, the remaining rightmost block of $\mathcal{Z}$ is consist of all zero columns after applying appropriate Gauss sweeping-out procedures to it further: $\mathcal{Z} \sim (\mathcal{Z}_1^*, O)$.

**More general case** The above cases can be extended to a general case with more than 3 fixed capitals. Inclusion of current goods does not change the formal framework.

The matrix $B - \lambda M$ is transformed into three blocks via nonsingular transformation of the form $P(B - \lambda M)Q$, where $P$ and $Q$ are nonsingular matrices.

Eventually, $B - \lambda M$ will be reduced to the following form:

$$B - \lambda M \sim \left(\begin{array}{c|ccc|c} \Psi(\lambda) & ** & ** & ** & \mathrm{O} \\ \hline & * & & & \\ \mathrm{O} & & \ddots & & \mathrm{O} \\ & & & * & \end{array}\right). \tag{2.2.2}$$

The leftmost columns with $\Psi(\lambda)$ constitute the block in which prices of brandnew commodities and the profit rate are determined. Okishio-Nakatani's goal of reduction was to solve $p\Psi(\lambda) = \mathbf{o}$, or, equivalently, $p = p(I - \Psi(\lambda))$, whilst, in the middle, the ratios of prices of aged fixed capital to brandnew fixed capital are determined. The remaining rightmost block is the zero block.

Remark that in the above reduction to the eigenvalue problem of $\Psi(\lambda)$, the desired $\lambda$ is determined implicitly. That is, in Sraffa-Okishio-Nakatani, the basic equation of prices of brandnew commodities is of the form $p = p(I - \Psi(\lambda))$, so that a nonnegative matrix $I - \Psi(\lambda)$ should have the Perron-Frobenius root equal to 1. From this, the profit rate is determined.[1]

As shown in the above, the matrix pencil is decomposed into three. Therefore, Okishio-Nakatani's reduction should be regarded as a decomposition of a matrix pencil of Marx-Sraffa price equilibrium. It is worth noting that $F$, $\mathcal{E}$, $\Lambda$ and $\mathcal{G}$ characterise the Sraffa-Okishio-Nakatani reduction procedure, that is based on SON1-4. If one of SONs, say SON4, is dropped, the reduction encounters difficulty.

Remark that a similar reduction can be applied to the system of processes to the system of quantities, as was done by Fujimori (1982).

[1] As for details, refer to *e.g.* Fujimori (1982).

# Chapter 3
# Renewal Dynamics of Fixed Capital

## 3.1 Introduction

In the preceding Chap. 2, it is shown that the right-side block of $\mathscr{Z}_1$ determines the relative ratios of prices of aged fixed capital to that of brand new fixed capital. In this chapter, it will be shown that this mechanism of determining relative prices of aged fixed capital is similar to the determination of values of fixed capital in renewal dynamics of fixed capital based on the companion matrix.

The problem is what happens if depreciation of fixed capital is reinvested immediately. That reinvestment of depreciation of fixed capital will bring about the stationary state of a larger capacity of production has been known as Marx-Engels effect or Ruchti-Lohmann effect. It seems that Kalecki (1954) and Steindl (1952) knew this effect, and they take into account this effect in discussing economic growth. Hence, we should call this effect Marx-Engles-Ruchi-Lohmann-Kalecki-Steindl effect, in short MERLKS. MERLKS is the target of study of renewal dynamics in this chapter.

We discuss two cases of depreciation. In one case, depreciation is taken into account while fixed capital is technically operable and operated in the production process; this case is often called the physical durability case, and regarded as the normal depreciation case. In the other case, depreciation is made in a period shorter than its durability, *i.e.*, in the accelerated mode.

In what follows, we *assume* that fixed capital is divisible ad infinitum. That is, fixed capital can be measured by real numbers. Mostly, we *assume* that the rate of depreciation is constant. We ignore the tax problem related to depreciation.



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## 3.2 Normal Depreciation

### *3.2.1 Yamada-Yamada and Markov Process*

Yamada and Yamada (1961) investigated renewal dynamics of fixed capital in terms of a linear difference equation.

Let $F,\ D,\ H$ and $K$ denote net investment of fixed capital, the amount of depreciation or amortisation, the amount of scrapped fixed capital and the nominal amount of fixed capital, respectively. Those are all functions of time $t$, which is a discrete variable.

The basic equations of renewal dynamics are then given by

$$D(t) = \frac{1}{m}K(t-1), \tag{3.2.1}$$
$$H(t) = F(t-m) + D(t-m), \tag{3.2.2}$$
$$K(t) = K(t-1) + F(t) + D(t) - H(t). \tag{3.2.3}$$

Yamada-Yamada assumes that net investment is expressed by:

$$F(t) = F_o(1+g)^t. \tag{3.2.4}$$

Then, one obtains

$$K(t) - \left(1 + \frac{1}{m}\right)K(t-1) + \frac{1}{m}K(t-m-1) = F(t) - F(t-m),$$

Hence,

$$K(t+m+1) - \left(1 + \frac{1}{m}\right)K(t+m) + \frac{1}{m}K(t) = F_o(1+g)^{t+1}((1+g)^m - 1). \tag{3.2.5}$$

The characteristic equation of the above is expressed by

$$\lambda^{m+1} - \left(1 + \frac{1}{m}\right)\lambda^m + \frac{1}{m} = 0. \tag{3.2.6}$$

After factorisation, one obtains:

$$(\lambda - 1)^2\left(\lambda^{m-1} + \frac{m-1}{m}\lambda^{m-2} + \cdots + \frac{1}{m}\right) = 0.$$

Therefore, characteristic roots other than duplicated 1 are derived from the following:

$$\lambda^{m-1} + \frac{m-1}{m}\lambda^{m-2} + \cdots + \frac{1}{m} = 0. \tag{3.2.7}$$

By applying Eneström-Kakeya's theorem on polynomial equations to (3.2.7), Yamada-Yamada proved that the dynamic path of fixed capital converges to a steady state, with diminishing oscillation.[1,2]

Now, consider the minimal polynomial of the companion matrix of (3.2.6), and one obtains:

$$\lambda^m - \left(1 - \frac{m-1}{m}\right)\lambda^{m-1} - \cdots - \left(\frac{2}{m} - \frac{1}{m}\right)\lambda - \frac{1}{m} = 0. \tag{3.2.8}$$

The companion matrix of (3.2.8) is represented by:

$$W_1 = \begin{pmatrix} \frac{1}{m} & \frac{1}{m} & \cdots & \frac{1}{m} & \frac{1}{m} \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{pmatrix}. \tag{3.2.9}$$

$W_1$ is a nonnegative primitive Markov matrix, and its left Perron-Frobenius vector $u$ corresponding to the Perron-Frobenius root 1 is strictly positive. Remark that the above companion matrix resembles the Leslie matrix of population dynamics.

The point is that $u$ gives the ratios of prices of fixed capital. That is, without loss of generality, one may put $u = (u_0, u_1, \dots, u_{m-1}) > \mathbf{0}$ with $u_0 = 1$. Then, $u_i = \dfrac{1}{m}u_0 + u_{i+1}$, *i.e.*, $u_{i+1} = u_i - \dfrac{1}{m}$. Hence, $u > \mathbf{0}$ such that $u(I - W_1) = \mathbf{0}$ is a vector of relative prices of fixed capital. Therefore, one sees the following: if the rate of depreciation is defined by the reciprocal of durability of fixed capital, there is a dynamic process, the companion matrix of which is a Markov matrix accompanying a vector of relative prices of fixed capital as its eigenvector.

In what follows, the above point will be generalised in the case with more general mode of amortisation; one of the dynamic process is specified, and that a relative price vector of fixed capital is an eigenvector corresponding to the Perron-Frobenius root of the companion matrix of the equation of renewal dynamics.

Let $u_i$ denote the relative ratio of age $i$ fixed capital to that of brandnew fixed capital and $u = \begin{pmatrix}1 & u_1 & \cdots & u_{m-1}\end{pmatrix}$ denote the relative price vector of fixed capital.

Before rewriting the equation of renewal dynamics, it is necessary to clarify definitions of depreciation coefficients.

Let $c_i$ and $d_i$ denote the rate of depreciation and the amount of depreciation of age $i$ fixed capital, respectively. Then, for $i = 0, 1, \dots, m-1$, one obtains

[1]As for Eneström-Kakeya's theorem, refer to *e.g.* Anderson et al. (1979).

[2]The characteristic equation of the original Yamada-Yamada model has duplicated root 1. Hence, the Jordan form of the companion matrix of (3.2.6) includes a Jordan block $\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}$; namely, the dynamics of the system is not free from the possibility of resonance.

$$\begin{aligned} d_0 + \cdots + d_{m-1} &= 1, \\ c_i u_i - d_i &= 0, \\ u_i - u_{i+1} &= d_i, \end{aligned}$$

where $d_0 = c_0$ and $c_{m-1} = 1$.

Remark $c_{m-1} = 1$ is equivalent to $u_m = 0$, and further, they are equivalent to

$$d_0 + d_1 + \cdots + d_{m-1} = u_0 = 1. \tag{3.2.10}$$

Now, transform Yamada-Yamada's fundamental equation into the one that is focused on gross investment of fixed capital $G(t)$.

$$G(t) = F(t) + D(t), \tag{3.2.11}$$
$$D(t) = d_0 G(t-1) + d_1 G(t-2) + \cdots + d_{m-1} G(t-m). \tag{3.2.12}$$

From this system of equations, one obtains the following[3]:

$$G(t) - d_0 G(t-1) - d_1 G(t-2) - \cdots - d_{m-1} G(t-m) = F(t). \tag{3.2.13}$$

The companion matrix of the characteristic equation of (3.2.13) is represented as follows:

$$W_2 = \begin{pmatrix} d_0 & d_1 & \cdots & d_{m-2} & d_{m-1} \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{pmatrix} \geq O.$$

Evidently, $W_2$ is a generalisation of $W_1$. It is easy to see that $u$ fulfills $u = uW_2$, and hence $u$ is one of left eigenvectors corresponding to the Perron-Frobenius root of $W_2$. Since $W_2$ is primitive, its Perron-Frobenius vector is unique except for scalar factors, so that the relative price vector of fixed capital $u$ is determined by $W_2$.

When the rate of depreciation is determined depending on the profit factor, then the form of a matrix $I - W_2$ corresponds to the right-side block of Okishio-Nakatani's decomposition which determines relative prices of aged fixed capital.

Thus, the structure of the above companion matrix corresponds to the middle block of $B - \lambda M$ in Chap. 2.

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[3] Or, equivalently, one obtains:

$$K(t) - d_0 K(t-1) - d_1 K(t-2) - \cdots - d_{m-1} K(t-m) = \sum_{\tau=0}^{m} F(t-\tau),$$

in view of

$$K(t) = G(t) + \cdots + G(t-m).$$

.

**Table 3.1** MERLKS effect: normal depreciation

| Period | | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| Capacity | | 1000 | 1200 | 1440 | 1728 | 2073.6 | 1488.32 | 1585.984 |
| Age | 0 | 1000 | 200 | 240 | 288 | 345.6 | 414.72 | 297.664 |
| | 1 | | 1000 | 200 | 240 | 288 | 345.6 | 414.72 |
| | 2 | | | 1000 | 200 | 240 | 288 | 345.6 |
| | 3 | | | | 1000 | 200 | 240 | 288 |
| | 4 | | | | | 1000 | 200 | 240 |
| Depreciation | | 200 | 240 | 288 | 345.6 | 414.72 | 297.664 | 317.1968 |

### 3.2.2 *Stationary State*

As seen in the above, renewal dynamics of fixed capital reveals a converging feature. With respect to the converging point of renewal dynamics in the above, for durability $m$, the stationary level of capacity is $\dfrac{2m}{m+1}$ to the original capacity 1 of 0-age fixed capital. This magnitude is obtained by computing labour values of fixed capital, both 0-age and aged.

In the middle of the converging process, the highest upsurge or spike of the oscillation takes place in the $m$-th period, and the greatest multiplier of capacity to the initial state then is $\left(1+\frac{1}{m}\right)^{m-1}$.

It must be noted that the stationary state and the greatest multiplier depend only on durability.

The next is an example of normal depreciation.

## 3.3 Accelerated Depreciation

So far, we discussed the case that fixed capital has predetermined durability and that depreciation is made in that given length of periods. For the sake of convenience, this case will be called the normal depreciation, if necessary.

In this section, we consider a case in which depreciation of fixed capital is made in the years shorter than its durability. *Assume* that durability of fixed capital is given by $m$ years and that depreciation is completed within $m-1$ years. This case should be called a case of accelerated depreciation. The case in this chapter is most simplest case. Remark that fixed capital is operated without depreciation, for free, at its final age.

Yamada-Yamada's system of equations of renewal dynamics will be transformed into the following. *We ignore new investment*: $F(t) = 0$.

Most of analyses of fixed capital depend on the assumption that the production processes proceed normally as expected ex ante. That is, fixed capital will be used until it reaches its materialistic durability. No economic durability, *i.e.*, old fixed capital is obliged to be scrapped in the face of the emergence of new technology.

Yamada-Yamada's system of equations will be replaced by the following:

$$H(t) = D(t-m), \tag{3.3.1}$$

$$K(t) = K(t-1) + D(t) - H(t), \tag{3.3.2}$$

$$D(t) = \frac{1}{\tau}\left(K(t) - D(t-m-1)\right). \tag{3.3.3}$$

From these, one gets, by putting $\tau' = \dfrac{1}{\tau - 1}$,

$$D(t) - \tau\tau' D(t-1) + \tau' D(t-m) + \tau' D(t-m-1) - \tau' D(t-m-2) = 0. \tag{3.3.4}$$

The characteristic equation of this is given by

$$\lambda^{m+2} - \frac{\tau}{\tau-1}\lambda^{m+1} + \frac{1}{\tau-1}\lambda^2 - \frac{1}{\tau-1}\lambda - \frac{1}{\tau-1} = 0. \tag{3.3.5}$$

This can be factorised as follows:

$$(\lambda - 1)^2\left(\lambda^m + \frac{\tau-2}{\tau-1}\lambda^{m-1} + \frac{\tau-3}{\tau-1}\lambda^{m-2} + \cdots + \frac{\tau-3}{\tau-1}\right) = 0. \tag{3.3.6}$$

The next example is a modification of the above Table 3.1 to show accelerated depreciation. We *assume* that fixed capital should operate for 5 periods, but depreciation will be made in 4 periods. The rate of depreciation is set to a constant, *i.e.* $\frac{1}{4}$. Hence, every equipment is operated in the last period of its life without keeping any economic value (Table 3.2).

Remark that fixed capital of age 4 is counted for as a part of capacity, but it has no value, and hence never enters into the object of depreciation any longer.

**Table 3.2** MERLKS effect with accelerated depreciation

| Period | | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| Capacity | | 1000 | 1250 | 1562.5 | 1953.125 | 2441.406 | 1488.32 | 1939.697 |
| Age | 0 | 1000 | 250 | 312.5 | 390.625 | 488.281 | 360.352 | 387.939 |
| | 1 | | 1000 | 250.0 | 312.500 | 390.625 | 488.281 | 360.352 |
| | 2 | | | 1000.0 | 250.000 | 312.500 | 390.625 | 488.281 |
| | 3 | | | | 1000.000 | 250.000 | 312.500 | 390.625 |
| | 4 | | | | | 1000.000 | 250.000 | 312.500 |
| Depreciation | | 250 | 312.5 | 390.625 | 488.281 | 360.352 | 387.939 | 406.799 |

As is easily seen from tables, accelerated depreciation yields a larger increase in capacity for longer periods than otherwise. Accelerated depreciation reveals to be something like positive externalities.

In the above case, in which the depreciation period is shorter than physical durability by 1, the multiplier of capacity is given by $2 = \frac{2(m-1)}{(m-1)+1} \cdot \frac{m}{m-1}$, irrespective of physical durability.

It seems that all of firms know, however, that their equipments will confront with new equipments embodying new technology. Hence, the accelerated amortisation method is often employed by firms, because most of firms are threatened by the advent of new technology.

## 3.4 Concluding Remarks

Several points with respect to fixed capital were discussed in this chapter.

Fixed capital is a materialistic basis of the system of commodity production. It exists in the production process more than two cycles of reproduction, as if it were the nature holding the long-term production project based on fixed capital. Its cost should be recovered, nonetheless, within durability. Because of this, fixed capital presents a property, not easy to deal with. As we saw in this chapter, depreciation has two faces. Is depreciation cost or profit? The answer might be 'both.'

It is one big issue left for economists to absorb the results obtained in the field of accounting. Specialists of accounting theory argue that individual firms are in disequilibrium while they are carrying out depreciation of fixed capital. According to Paton and Littleton (1940), it is not certain that depreciation can be done as planned until fixed capital is worn out and scrapped as planned.

When new investment of a unit of fixed capital is made by means of brandnew fixed capital alone, the equilibrium level of capacity of production should be $\frac{2m}{m+1}$ for durability $m$, so that the whole economy are not in equilibrium.

If we consider an economy in which new types of fixed capital emerge, the initial investment of fixed capital creates disequilibrium in the economy. Although renewal dynamics yields convergence, indeed, but the velocity of convergence matters; it is very slow. In our simulation, the period necessary for convergence should be at least more than 4 times as long as durability. Before its convergence, the dynamic process of changing capacity of fixed capital will encounter several spikes or upsurges, and falls. Thus, fixed capital is an important disequilibrium factor of the commodity production system.

# Chapter 4
# Profit and Growth in the SON Economy

## 4.1 Introduction

This chapter first examines the problem of reproduction including fixed capital, both the uniform rates of profit and growth based on the equality system. The equilibrium production price and the equilibrium quantity system frameworks including fixed capital, which will be introduced below, have the prerequisite of prepaid wages; from this, it is within the Marxian paradigm. On the other hand, analyzing and examining the problem of fixed capital from the angle of joint production is within the Sraffian paradigm. Therefore, we named the model as a narrowly defined Marx-Sraffa model.

In fact, the rate of profit and the rate of accumulation defined by net profit do not satisfy the Cambridge equation (Fujimori and Li 2010) under this subsystem. The second half of this chapter clearly specifies how to define the rate of profit and rate of accumulation so as to re-establish the Cambridge equation.

## 4.2 Framework of Analysis

### *4.2.1 The Reproduction Model with Only 1 Type of Fixed Capital*

First, the issue of equilibrium production price and equilibrium growth is examined with the presence of fixed capital. Here, the simplest analytical framework is adopted: only fixed capital is included as production information, and mobile production elements such as raw materials are ignored from the calculations.

The cycle of reproduction is calculated in years. The labour force is assumed to be homogenous. There is a 1 type of fixed capital for brand new commodities, and its durability is 3 years. It is *assumed* that fixed capital has the same efficiency irrespective of ages, and once it reaches 3 years old, it is scrapped without cost. If



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fixed capital of the different years of age is seen as different commodities, there are in fact 3 types of fixed capital in the economy. The fixed capital of 0, 1 and 2 year(s) of age is labeled, respectively, as commodity 1, 2 and 3. If the efficiency of fixed capital of all ages is same, so is the input coefficient of fixed capital.

Assuming that only fixed capital and labour are required for the production of fixed capital, let $k$ and $l$ denote the input coefficient of fixed capital and labour, respectively.

When the production process formed by fixed capital of age 0 and labour is operated, a part of output is aged fixed capital of age 1. If the activity of the process is normalised with respect to the unit of brand new commodities, the following 3 types of production processes would coexist, that is,

$$\begin{pmatrix} k \\ 0 \\ 0 \\ l \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ k \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ k \\ 0 \\ l \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \\ k \end{pmatrix}; \begin{pmatrix} 0 \\ 0 \\ k \\ l \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

The input-output relationship related to production of fixed capital and shifts of processes are shown in Table 4.1. $x^j$ indicates the activity level of process $j$.

### 4.2.2 The Model of 4 Commodities from 2 Sectors

We apply the above framework to an economy with 2 sectors; sector 1 produces brand new fixed capital as its main products, and sector 2 does consumer goods.

**Table 4.1** Input-output relationship in production of fixed capital

| | | Inputs | | | Outputs | | |
|---|---|---|---|---|---|---|---|
| Production process | | 1 | 2 | 3 | 1 | 2 | 3 |
| Period t | Age 0 | $kx^1$ | | | $x^1$ | | |
| | Age 1 | | | | $kx^1$ | | |
| | Age 2 | | | | | | |
| Period $t+1$ | Age 0 | $kx^2$ | | | $x^2$ | $x^1$ | |
| | Age 1 | | $kx^1$ | | $kx^2$ | | |
| | Age 2 | | | | | $kx^1$ | |
| Period $t+2$ | Age 0 | $kx^3$ | | | $x^3$ | $x^2$ | $x^1$ |
| | Age 1 | | $kx^2$ | | $kx^3$ | | |
| | Age 2 | | | $kx^1$ | | $kx^2$ | |

Raw materials are ignored.[1] Both sectors have 3 production processes. As mentioned earlier, the fixed capital of 0, 1 and 2 years of age is labeled respectively as commodity 1, 2 and 3. Consumer goods are labeled as commodity 4.

The input coefficient and the labour input are denoted by $k_i$ and $l_i$, respectively, where $i = 1, 2$ indicates sectors concerned. The input matrix $A$, labour vector $L$, output matrix $B$ of processes in sector 1 and 2, and wage goods bundle $f$ can be expressed as follows:

$$A = \begin{pmatrix} k_1 & 0 & 0 & k_2 & 0 & 0 \\ 0 & k_1 & 0 & 0 & k_2 & 0 \\ 0 & 0 & k_1 & 0 & 0 & k_2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{4.2.1}$$

$$L = \begin{pmatrix} l_1 \; l_1 \; l_1 \; l_2 \; l_2 \; l_2 \end{pmatrix}, \tag{4.2.2}$$

$$B = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ k_1 & 0 & 0 & k_2 & 0 & 0 \\ 0 & k_1 & 0 & 0 & k_2 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}, \tag{4.2.3}$$

$$f = \begin{pmatrix} 0 \\ 0 \\ 0 \\ b \end{pmatrix}. \tag{4.2.4}$$

### 4.2.3 *The Equilibrium of Production Prices and the Level of Activities*

Let $p = (p_{10} \; p_{11} \; p_{12} \; p_2)$ and $\pi$ denote the price vector and the rate pf profit, respectively. Put $\omega = p\boldsymbol{f}$, and the equilibrium system of production price is expressed by

$$p\boldsymbol{B} = (1 + \pi)(p\boldsymbol{A} + \omega\boldsymbol{L}). \tag{4.2.5}$$

We now examine the status of economic growth at an equal growth rate. As indicated by $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{L}$, each column corresponds to one production process, and 6 production processes exist. Starting from the left, it was labeled in sequence as process 1, 2, 3, 4, 5 and 6. In 3 periods after generating the first process, these production processes will coexist. In terms of the generative process of the production process, after process 1 is generated by investment, processes 2 and 3 will be generated in turn. The generative relationships of process 4, 5 and 6 follow the same logic. Therefore,

[1] Although these sectors jointly produce aged fixed capital, brand new fixed capital and consumer goods are the main products. Therefore, in this sense, we named them as the fixed capital production sector and the consumer goods production sector.

as long as process 1 and 4 are generated, process 2, 3 and processes 5, 6 will be generated automatically.

Let the activity vector be expressed by

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_6 \end{pmatrix}.$$

Let $u$ and $g$ denote the nonproductive consumption vector and the growth rate, respectively, and

$$\boldsymbol{B}x = (1+g)(\boldsymbol{A} + \boldsymbol{f}\boldsymbol{L})x + \boldsymbol{u}. \tag{4.2.6}$$

### 4.2.4 The Reduced Price System

In this subsection, the simple model discussed above will be taken up so as to illustrate the issue of determining the equilibrium in SON, which is discussed in Chap. 2.

Let $p_{1j}$ denote the price of fixed capital of age $j$. Remark that $p_1 = p_{10}$. The equilibrium of production prices is expressed as follows:

$$p_1 + p_{11}k_1 = (1+\pi)(p_1k_1 + \omega l), \tag{4.2.7}$$

$$p_1 + p_{12}k_1 = (1+\pi)(p_11k_1 + \omega l), \tag{4.2.8}$$

$$p_1 = (1+\pi)(p_{12}k_1 + \omega l), \tag{4.2.9}$$

$$p_2 + p_{11}k_2 = (1+\pi)(p_1k_2 + \omega l), \tag{4.2.10}$$

$$p_2 + p_{12}k_2 = (1+\pi)(p_11k_2 + \omega l), \tag{4.2.11}$$

$$p_2 = (1+\pi)(p_{12}k_2 + \omega l). \tag{4.2.12}$$

From (4.2.7)–(4.2.9) we eliminate $p_{11}$ and $p_{12}$, and it follows that

$$p_1 = (1+\pi)\omega l + (\varphi(\pi) + \pi)p_1k_1.$$

The coefficient part $\varphi(\pi)$ of the fixed capital cost on the right-hand side is called the rate of depreciation. The rate of depreciation depends on the residual years of depreciation. To be more specific, if the residual years of depreciation are defined as $\tau$, one obtains the formula

$$\varphi(\pi, \tau) = \frac{1}{\sum_{s=0}^{\tau-1}(1+\pi)^s}. \tag{4.2.13}$$

It is clear that $\varphi$ is a decreasing function of $\pi$. The equilibrium price of aged fixed capital is

$$p_{11} = (1 - \varphi(\pi, 3))p_1,\ p_{12} = (1 - \varphi(\pi, 2))p_{11}. \tag{4.2.14}$$

In this way, the rate of depreciation is determined, dependent on the rate of profit, and the price ratio of fixed capital of different ages will be determined depending on the rate of depreciation.

The equation determining the equilibrium price of aged fixed capital (4.2.14) has important implications. That is,

(1) the rate of depreciation is dependent on the rate of profit,
(2) the determination of prices of aged fixed capital *via* the rate of depreciation corresponds to the middle block of the reduction in (2.2.2), in Chap. 2, p. 12.

We illustrated in the above that a joint production system can be reduced to the system of non-joint production. That is, we define

$$A_o = \begin{pmatrix} k_1 & k_2 \\ 0 & 0 \end{pmatrix},\ l = \begin{pmatrix} l_1 & l_2 \end{pmatrix}.$$

Putt $\bar{p} = (p_1\ p_2)$, and it follows that

$$\bar{p} = \bar{p}(\pi A_o + \bar{\phi}(\pi)A_o + (1 + \pi)fl),$$

where $\bar{\varphi}(\pi) = \begin{pmatrix} \varphi(\pi, \tau) & 0 \\ 0 & 1 \end{pmatrix}$, with durability of fixed capital $\tau$.

The part of the coefficient matrix can be summarized and expressed as follows:

$$K(\pi) = (1 + \pi)fl + (\pi I + \bar{\varphi}(\pi))\, A_o, \tag{4.2.15}$$

and one has

$$\bar{p} = \bar{p}K(\pi). \tag{4.2.16}$$

In addition, $\pi$ can be determined by the following equation:

$$|I - K(\pi)| = 0. \tag{4.2.17}$$

Remark that elements of $K(\pi)$ are increasing functions of $\pi$:

With regard to the question of determining the uniform rate of profit, only the system of brand new commodities needs to be considered for determining the uniform rate of profit, once the rate of depreciation is determined. (*Cf.* Chap. 2.)

### 4.2.5 Reduced Quantity System

As in the previously illustrated reduction process of the production price equilibrium, formula (4.2.6) can be reduced to a system where only brand new commodities are included.

In fact, (4.2.6) can be expressed as below:

$$x_1 + x_2 + x_3 = (1+g)(k_1 x_1 + k_2 x_4), \tag{4.2.18}$$

$$k_1 x_1 + k_2 x_4 = (1+g)(k_1 x_2 + k_2 x_5), \tag{4.2.19}$$

$$k_1 x_2 + k_2 x_5 = (1+g)(k_1 x_3 + k_2 x_6), \tag{4.2.20}$$

$$x_4 + x_5 + x_6 = (1+g)b(l_1(x_1 + x_2 + x_3) + l_2(x_4 + x_5 + x_6)) + C. \tag{4.2.21}$$

Note that one has

$$\begin{aligned}(1+g)^2(x_1 + x_2 + x_3) &= (1+g)^2(k_1 x_1 + k_2 x_4),\\ (1+g)(k_1 x_1 + k_2 x_4) &= (1+g)^2(k_1 x_2 + k_2 x_5),\\ k_1 x_2 + k_2 x_5 &= (1+g)(k_1 x_3 + k_2 x_6).\end{aligned}$$

If we *assume* that the economy is growing at the uniform rate $g$,

$$\begin{aligned}x_2 &= (1+g)^{-1}x_1,\ x_3 = (1+g)^{-2}x_1,\\ x_5 &= (1+g)^{-1}x_1,\ x_6 = (1+g)^{-2}x_4,\end{aligned}$$

and put

$$\begin{aligned}q_1 &= x_1 + x_2 + x_3,\\ q_2 &= x_4 + x_5 + x_6,\end{aligned}$$

then one can eliminate the activity level $x_j$ from the system. Thus, one can obtain the system of equations with respect to output. Let $q = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$ and $\boldsymbol{u} = \begin{pmatrix} 0 \\ C \end{pmatrix}$, denote the output vector and nonproductive consumption vector, respectively, and one obtains

$$q = (\bar{\varphi}(g, \tau) + gI)\, A_o q + (1+g)\, f l q + \boldsymbol{u}, \tag{4.2.22}$$

$\bar{\varphi}(g, \tau)$ in the equation is called the renewal coefficient.

Although the renewal coefficient and the rate of depreciation look nominally different, they stems from the identical function $\varphi$. The difference lies in the meaning of the first argument of the function.

Since the coefficient matrix of the reduced system of equations is a nonnegative matrix, the Perron-Frobenius theorem can be applied to the existence of equilibrium.

Remark that it is also possible to eliminate $p_1$ and $p_{11}$ from the above. The same idea can be applied to the quantity side.

## 4.3 The Cambridge Equation in SON

### *4.3.1 Some Preparations*

As for the augmented input matrix $M$ and output matrix $B$,

$$M = \begin{pmatrix} k_1 & 0 & 0 & k_2 & 0 & 0 \\ 0 & k_1 & 0 & 0 & k_2 & 0 \\ 0 & 0 & k_1 & 0 & 0 & k_2 \\ bl_1 & bl_1 & bl_1 & bl_2 & bl_2 & bl_2 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ k_1 & 0 & 0 & k_2 & 0 & 0 \\ 0 & k_1 & 0 & 0 & k_2 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}.$$

In SON, the basic coefficients indicating the input-output relationship are

$$K = \begin{pmatrix} k_1 & k_2 \\ 0 & 0 \end{pmatrix}, \bar{\psi}(r,\tau) = \begin{pmatrix} \psi(r,\tau) & 0 \\ 0 & 1 \end{pmatrix}, f = \begin{pmatrix} 0 \\ b \end{pmatrix}, l = \begin{pmatrix} l_1 & l_2 \end{pmatrix}.$$

It is well known that in the no-joint production system without fixed capital, the Cambridge equation indicating the relationship between the profit rate and growth rate is established.

Now, under the system of joint production including aged fixed capital, total output can be expressed as

$$pBx = (1+\pi)pMx, \tag{4.3.1}$$

$$pBx = (1+g)pMx + p\boldsymbol{u}. \tag{4.3.2}$$

It is easily seen that

$$\pi pMx = gpMx + p\boldsymbol{u}. \tag{4.3.3}$$

The magnitude of “total output − costs” is tentatively the total profit $\Phi_1$; and one sees from (4.3.1) that

$$\Phi_1 = \pi pMx.$$

whereas, the rate of nonproductive consumption of capitalists is denoted by $c$, so that the rate of accumulation $\alpha$ can be defined as follows:

$$\alpha = 1 - c = 1 - \frac{p\boldsymbol{u}}{\pi pMx}. \tag{4.3.4}$$

**Table 4.2** P/L: Joint production case

| | |
|---|---|
| $pAx$ | $pBx$ |
| $pf\ell$ | |
| $pBx - pMx$ | |

Therefore, the Cambridge equation is established.[2] That is,

$$g = \alpha\pi. \tag{4.3.5}$$

The Cambridge equation on the relationship between the profit and growth rates seems thus established.

The problem, however, is what the contents of this equation in this case.

If $g = \pi$, there will be no deviation between the rate of depreciation and the rate of renewal. In this case, the amount of depreciation is identical with the amount of renewal investment. The Cambridge equation, that the product of the rate of accumulation $\alpha(= 1)$, and the rate of profits $\pi$ equals to the rate of growth $g$, is established. The same goes for the SON economy obtained from reduction.

If the existence of nonproductive consumption is considered, then, in general, $g < \pi$, so that the rate of renewal is larger than the rate of depreciation. In the Marx-Sraffa system, the difference between $\varphi(g)$ and $\varphi(\pi)$ will reveal itself.

Viewing the Marx-Sraffa joint production system through the format of the profit-and-loss statement (P/L), it can be expressed in a simple way as below:

From this table, one sees that $pBx - pMx$ part can be different from profits obtained in the no-joint production system. The rate of profit calculated from the joint production system is a kind of "gross" rate of profit. The difference of $pBx - pMx$ includes the amount of depreciation, so that it is included on the left side of Table 4.2, and adding net profit turns it into gross profit. The profit part demanded in the above table is gross profit. The rate of profit gained from the joint production system is the rate of gross profit.

Since SON is a reduction of a joint production system, $\pi$ and $g$ of SON must reflect those of the original joint production system. Both profit and accumulation require being redefined on the gross base, such as "gross" profit and "gross" accumulation.

### 4.3.2 The Cambridge Equation of the SON Economy

The price equation comprises of only new commodities, that is,

$$\bar{p} = \bar{p}\bar{\psi}(\pi, \tau)K + \pi\bar{p}K + (1 + \pi)\bar{p}fl,$$

[2] In fact, it can be seen from (4.3.2) that the total profit can be expressed as $gpMx + p\boldsymbol{u}$. This is used to define the rate of accumulation $\alpha$, that is, $\alpha = 1 - c = 1 - \dfrac{p\boldsymbol{u}}{gpMx + p\boldsymbol{u}}$, which will also generate results identical to those of (4.3.5).

and from the quantity equation (4.2.22), it is known that the output amount is

$$\bar{p}q = \bar{p}\bar{\psi}(\pi, \tau)Kq + \pi\bar{p}Kq + (1+\pi)\bar{p}flq, \tag{4.3.6}$$

$$\bar{p}q = \bar{p}\bar{\psi}(g, \tau)Kq + g\bar{p}Kq + (1+g)\bar{p}flq + \bar{p}C, \tag{4.3.7}$$

Further, one can obtain

$$\pi\bar{p}(K + fl)q + \bar{p}\bar{\psi}(\pi, \tau)Kq = g\bar{p}(K + fl)q + \bar{p}\bar{\psi}(g, \tau)Kq + \bar{p}C. \tag{4.3.8}$$

This equation indicates the following relationship:

Net profit + Depreciation cost = Total investment + Consumption,

where Total investment = Net investment + Renewal investment.

In other words, the following perspective is required to consider the various relationships above:

Gross profit = Net profit + Depreciation cost,

Gross accumulation = Net investment + Renewal investment.

Adopt the following definitions to the rate of gross profit and the rate of gross accumulation:

Rate of gross profit = Net profit ÷ Amount of capital at beginning of period,

and

Rate of gross accumulation = Gross accumulation ÷ Gross profit.

Assume that the amount of capital at the beginning of period $t$ is denoted as $K(t)$, and the amount of capital at the end of period $t$, which is the beginning of period $t+1$, is $K(t+1)$. Then, the amount of capital increased in period $t$ is $gK(t)$, that is,[3]

$$\Delta K(t) = K(t+1) - K(t) = gK(t). \tag{4.3.9}$$

The economy is assumed to grow at the uniform rate $g$, so that

$$\Delta K(t) = k_1x_1^1 + k_2x_2^1 + g(k_1x_1^1 + k_2x_2^1).$$

[3] For similar analyses, refer to Koshimura (1967, Chap. 5).

This can be rewritten as

$$\Delta K(t) = \frac{g(1+g)^3}{(1+g)^3 - 1}(k_1 q_1 + k_2 q_2).$$

From (4.3.9), one can obtain

$$K(t) = \frac{1}{g}\Delta K(t) = \frac{(1+g)^3}{(1+g)^3 - 1}(k_1 q_1 + k_2 q_2).$$

Hence, the total amount of capital at the beginning of the period is

$$\frac{(1+g)^3}{(1+g)^3 - 1}(p_1 k_1 q_1 + p_1 k_2 q_2) = \frac{1}{g}(g\bar{p}Kq + \psi(g,3)\bar{p}Kq).$$

Therefore, one sees that $\pi$ and $\alpha$ are evaluated, respectively, by

$$\pi = \frac{\pi\bar{p}Kq + \psi(\pi,3)\bar{p}Kq}{\frac{1}{g}(g\bar{p}Kq + \psi(g,3)\bar{p}Kq)}, \tag{4.3.10}$$

and

$$\alpha = \frac{g\bar{p}Kq + \psi(g,3)\bar{p}Kq}{\pi\bar{p}Kq + \psi(\pi,3)\bar{p}Kq}. \tag{4.3.11}$$

Further,

$$\alpha\pi = \frac{g\bar{p}Kq + \psi(g,3)\bar{p}Kq}{\pi\bar{p}Kq + \psi(\pi,3)\bar{p}Kq}\,\frac{\pi\bar{p}Kq + \psi(\pi,3)\bar{p}Kq}{\frac{1}{g}(g\bar{p}Kq + \psi(g,3)\bar{p}Kq)} = g.$$

Thus, *the Cambridge equation is reestablished.*

## 4.4 Concluding Remarks

If the existence of aged fixed capital is clearly specified, depreciation as a concept may not matter. If aged fixed capital is eliminated from the system of joint production, however, the calculation of depreciation becomes indispensable; depreciation substitutes aged fixed capital in the sense that prices of aged fixed capital can be evaluated by means of the rate of depreciation. The price of aged fixed capital is related to the relative price and the rate of profit in the overall economy, and thus, the rate of depreciation used in the calculation of aged fixed capital is also related to the rate of profit.

In this chapter, we illustrated the reduction of the joint production system to SON, in both price and quantity sides, with a very simple model. We demonstrated in that simple model that the Cambridge equation that bridges profits and growth by extending the contents of profit to the internal reserve, and that of accumulation to gross investment.

We add here some comments on SON and the input-output tables. The input-output tables include brand new commodities produced in a period concerned, with entry of durable capital consumed in that period. Hence, the input-output tables are very close to SON. In obtaining the information of economic indices from input-output tables, it will be important to recognise the close connection between the input-output tables and SON.

# Chapter 5
# Economic Durability and Hardening Effects of Fixed Capital

## 5.1 Introduction

One of the features of fixed capital is that it is operated beyond a cycle of reproduction of the economy. The first problem that should be raised with respect to fixed capital is how long, or how many periods, it can be operated.

In the field of economics, durability of fixed capital is conventionally treated as determined physically; moreover, the efficiency of fixed capital is taken as a constant and is continued to be used until its life is over.

Sraffa (1960), Okishio and Nakatani (1975), Abraham-Frois and Berrebi (1979), Fujimori (1982), Schefold (1989), Schefold (1997) Nakatani (1994), Kurz and Salvadori (1995) and Fujimori and Li (2010) adopted this stance while discussing problems of fixed capital.

There is another approach toward determining durability of fixed capital. That is, the efficiency of fixed capital declines as it is operated. If fixed capital is aged and its efficiency becomes too low to raise any profits by operating it, then it will be scrapped. Thus, durability of fixed capital can be determined endogenously.

In this chapter, we discuss the endogenous determination of durability of fixed capital with declining efficiency, in the framework of linear inequalities by means of numerical examples. The theory basis for computation depends on Fujimoto (1975).

## 5.2 The Model

An economy, which consists of one kind of fixed capital and one kind of consumption good, is assumed. Suppose that fixed capital is employed for $n$ years tentatively and its efficiency declines by the rate of $\alpha$ in every year, and the economy has a long-run steady balanced growth.


B. Li, *Linear Theory of Fixed Capital and China's Economy*,
DOI 10.1007/978-981-10-4065-8_5

**Table 5.1** The basic production structure of fixed capital ($i = 1$)

| Processes | Input | | | | | Output | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | ... | $n-1$ | $n$ | 1 | 2 | ... | $n-1$ | $n$ |
| 0-year-old | $k_1$ | | | | | 1 | $\alpha$ | ... | $\alpha^{n-2}$ | $\alpha^{n-1}$ |
| 1-year-old | | $k_1$ | | | | $k_1$ | | | | |
| 2-year-old | | | $k_1$ | | | | $k_1$ | | | |
| $\vdots$ | | | | $\ddots$ | | | | $\ddots$ | | |
| $(n-1)$-years-old | | | | | $k_1$ | | | | $k_1$ | 0 |
| Labour | $fl_1$ | $fl_1$ | ... | $fl_1$ | $fl_1$ | 0 | 0 | ... | 0 | 0 |

Let $k_i$ denote the input of fixed capital for goods $i$, $l_i$ the labour input to goods $i$, $i = 1, 2$, and $f$ the real wage rate. Then, the basic production structure of fixed capital will be shown as in Table 5.1.

Let $A$, $B$, $L$ and $F$ denote the input- and output matrices, the labour input, and the wage-goods bundle, respectively. They are given as follows:

$$A = \begin{pmatrix} k_1 & & & & k_2 & & & \\ & k_1 & & & & k_2 & & \\ & & \ddots & & & & \ddots & \\ & & & k_1 & & & & k_2 \\ 0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 \end{pmatrix}, \tag{5.2.1}$$

$$L = \big(l_1\ l_1\ \ldots\ l_1\ l_2\ l_2\ \ldots\ l_2\big), \tag{5.2.2}$$

$$F = {}^t\big(0\ 0\ \ldots\ 0\ f\big), \tag{5.2.3}$$

$$B = \begin{pmatrix} 1 & \alpha & \ldots & \alpha^{n-1} & 0 & \ldots & 0 & 0 \\ k_1 & & & & k_2 & & & \\ & \ddots & & & & \ddots & & \\ & & k_1 & & & & k_2 & \\ 0 & \ldots & 0 & 0 & 1 & \alpha & \ldots & \alpha^{n-1} \end{pmatrix}. \tag{5.2.4}$$

Let $r$ stand for the uniform profit rate in the whole economy. Then, the linear programming problem that maximizes the wage rate $w = pF$ is as follows.

$$\max\{pF \mid \frac{1}{1+r} pB \leqq pA + L,\ p \geq \mathbf{0}\}. \tag{5.2.5}$$

The constraint condition in (5.2.5) represents that any gain of processes is not more than normal profits. (See Fujimoto 1975.)

Let $g$ and $x$ stand for the uniform growth rate in the whole economy and the activity level of its production processes, respectively. If $r = g$, the dual problem of the standard maximization problem (5.2.5) is expressed as follows:

$$\min\{Lx \mid \frac{1}{1+g}Bx \geqq Ax + F, x \geq \mathbf{0}\}. \tag{5.2.6}$$

The dual problem (5.2.6) means that the amount of labour input must be minimised on the premise that supply is not less than demand.

## 5.3 Simulation of the Determination of Economic Durability

### 5.3.1 The Numerical Value Settings

First, normalise the real wage rate $f = 1$; $A$, $L$ and $B$ are set up as follows.

$$A = \begin{pmatrix} 0.80 & & & & 0.70 & & & \\ & 0.80 & & & & 0.70 & & \\ & & \ddots & & & & \ddots & \\ & & & 0.80 & & & & 0.70 \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \end{pmatrix}, \tag{5.3.1}$$

$$L = \begin{pmatrix} 0.25 \; 0.25 \dots 0.25 \; 0.35 \; 0.35 \dots 0.35 \end{pmatrix}, \tag{5.3.2}$$

$$B = \begin{pmatrix} 1 & \alpha & \dots & \alpha^{n-1} & 0 & \dots & 0 & 0 \\ 0.80 & & & & 0.70 & & & \\ & \ddots & & & & \ddots & & \\ & & 0.80 & & & & 0.70 & \\ 0 & \dots & 0 & 0 & 1 & \alpha & \dots & \alpha^{n-1} \end{pmatrix}, \tag{5.3.3}$$

where $0 < \alpha < 1$ denotes the magnitude of decrease in the efficiency of fixed capital.

### 5.3.2 The Standard Maximum Problem and Production Price Ratios

First, let $\alpha$ be fixed. If $r$ is given, then a linear programming problem (5.2.5) is solved. Observing the optimal solution $x^i$ the activity level of the non-operated process is given by zero. From this result, the number of years following which fixed capital is discarded can be determined. Namely, durability of fixed capital $\tau$ is determined depending on $r$. Here, if the standard maximum problem (5.2.5) is solved under the following given conditions:

$$\alpha = 0.90,\ r = 0.20,\ n = 20, \tag{5.3.4}$$

then the production price ratio $p$ is obtained as follows.

$$p = \big(0.4868\ 0.2979\ 0.1598\ 0.0740\ 0.0202\ 0\ \ldots\ 0\ 0.6204\big) \tag{5.3.5}$$
$$\propto \big(1\ 0.6120\ 0.3284\ 0.1519\ 0.0415\ 0\ \ldots\ 0\ 1.2745\big) \tag{5.3.6}$$

Hence, it turns out that the endogenously determined durability of fixed capital $\tau$ is 5 years. The maximum nominal wage $w_{max}$ obtained by the optimal solution is as follows.

$$w_{max} = pF = 0.6204. \tag{5.3.7}$$

Therefore, when we evaluate the real wage rate $f^*$ in accordance with the price of the brand-new fixed capital $p^0$, it is obtained as follows.

$$f^* = \frac{w_{max}}{p^0} = \frac{0.6204}{0.4868} = 1.2745. \tag{5.3.8}$$

The result of the simulation for the endogenous determination of the durability in the variable efficiency of fixed capital is shown in Table 5.2.

Figure 5.1 shows the three-dimensional relationship that exists among the profit rate $r$, efficiency $\alpha$, and durability of fixed capital $\tau$.

### *5.3.3 The Activity Level of Production Processes*

In the same way, when the dual problem (5.2.6) is solved under condition (5.3.4), the activity level in the two production processes can be obtained.

$$x = {}^t(0, 0, 0.1375, 0.2906, 0.2422, 0, \ldots, 0, 0.5740, 0.4783, 0.2415, 0, \ldots, 0) \tag{5.3.9}$$

In (5.3.9), it can be confirmed that fixed capital of 2-year-old, 3-year-old and 4-year-old are in operation in the production process of fixed capital, and also that fixed capital of 0-year-olds (brand-new goods), 1-year-olds and 2-year-olds are in operation in the production process of consumption goods. This result is consistent with the equation of the ratios of production price (5.3.5). Naturally, the optimal solution $u$ in this case, in accordance with the one for standard maximum problems, is as follows.

**Table 5.2** The endogenous determination of the durability in the variable efficiency

| $r$ | $\alpha = 0.95$ | | $\alpha = 0.90$ | | $\alpha = 0.85$ | | $\alpha = 0.80$ | | $\alpha = 0.75$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $f^*$ | $\tau$ | $f^*$ | $\tau$ | $f^*$ | $\tau$ | $f^*$ | $\tau$ | $f^*$ | $\tau$ |
| 0.05 | 1.3681 | 5 | 1.3137 | 4 | 1.2863 | 3 | 1.2389 | 3 | 1.2394 | 3 |
| 0.10 | 1.3650 | 6 | 1.3004 | 4 | 1.3161 | 3 | 1.2243 | 3 | 1.2247 | 3 |
| 0.15 | 1.3434 | 6 | 1.2880 | 5 | 1.3092 | 4 | 1.2093 | 4 | 1.2090 | 3 |
| 0.20 | 1.3036 | 7 | 1.2745 | 5 | 1.2055 | 4 | 1.1963 | 4 | 1.1924 | 3 |
| 0.25 | 1.2899 | 8 | 1.2598 | 5 | 1.1919 | 5 | 1.1822 | 4 | 1.1572 | 3 |
| 0.30 | 1.2750 | 9 | 1.2451 | 6 | 1.1785 | 5 | 1.1672 | 4 | 1.1591 | 4 |
| 0.35 | 1.2319 | 10 | 1.1951 | 6 | 1.1640 | 5 | 1.1513 | 4 | 1.0611 | 4 |
| 0.40 | 1.2156 | 11 | 1.1671 | 7 | 1.1487 | 6 | 1.1354 | 5 | 1.0487 | 4 |
| 0.45 | 1.1983 | 12 | 1.1516 | 8 | 1.1334 | 6 | 1.0597 | 5 | 1.0362 | 5 |
| 0.50 | 1.1595 | 13 | 1.1356 | 8 | 1.1171 | 6 | 1.0464 | 6 | 1.0236 | 5 |
| 0.55 | 1.1415 | 15 | 1.1187 | 9 | 1.0589 | 7 | 1.0331 | 6 | 1.0105 | 5 |
| 0.60 | 1.1228 | 16 | 1.0734 | 10 | 1.0443 | 7 | 1.0192 | 6 | 0.9969 | 6 |
| 0.65 | 1.1036 | 18 | 1.0572 | 11 | 1.0293 | 8 | 1.0048 | 7 | 0.9832 | 6 |
| 0.70 | 1.0710 | 20 | 1.0404 | 11 | 1.0137 | 9 | 0.9901 | 7 | 0.9689 | 6 |
| 0.75 | 1.0522 | 22 | 1.0233 | 13 | 0.9976 | 9 | 0.9748 | 8 | 0.9543 | 7 |
| 0.80 | 1.0331 | 24 | 1.0057 | 14 | 0.9812 | 10 | 0.9591 | 9 | 0.9392 | 8 |
| 0.85 | 1.0138 | 26 | 0.9877 | 15 | 0.9643 | 11 | 0.9431 | 10 | 0.9238 | 9 |
| 0.90 | 0.9944 | 29 | 0.9695 | 17 | 0.9470 | 13 | 0.9266 | 11 | 0.9079 | 10 |
| 0.95 | 0.9748 | 33 | 0.9510 | 19 | 0.9294 | 14 | 0.9097 | 13 | 0.8916 | 13 |

$$u_{min} = Lx = 0.6204 (= w_{max}). \tag{5.3.10}$$

The output ratio $q = Bx$ is shown as follows:

$$q = {}^t\big(0.4821\ 0.4018\ 0.3348\ 0.2790\ 0.2325\ 0.1938\ 0 \ldots 0\ 1.2\big) \tag{5.3.11}$$
$$\propto {}^t\big(1\ 0.8333\ 0.6944\ 0.5787\ 0.4823\ 0.4019\ 0 \ldots 0\ 2.4890\big). \tag{5.3.12}$$

## 5.4 Simulation of the Hardening Effect of Fixed Capital

### *5.4.1 Numerical Settings*

Let the efficiency of fixed capital $\alpha$ be variable according to its age. The efficiency of aged fixed capital such as 0-year-olds, 1-year-old, 2-year-old ... is set up as follows:

$$\alpha = 1,\ \beta,\ \beta,\ 1,\ 0.95,\ 0.9,\ 0.85,\ 0.8,\ 0.75,\ 0.7,\ 0.65,\ \ldots \tag{5.4.1}$$

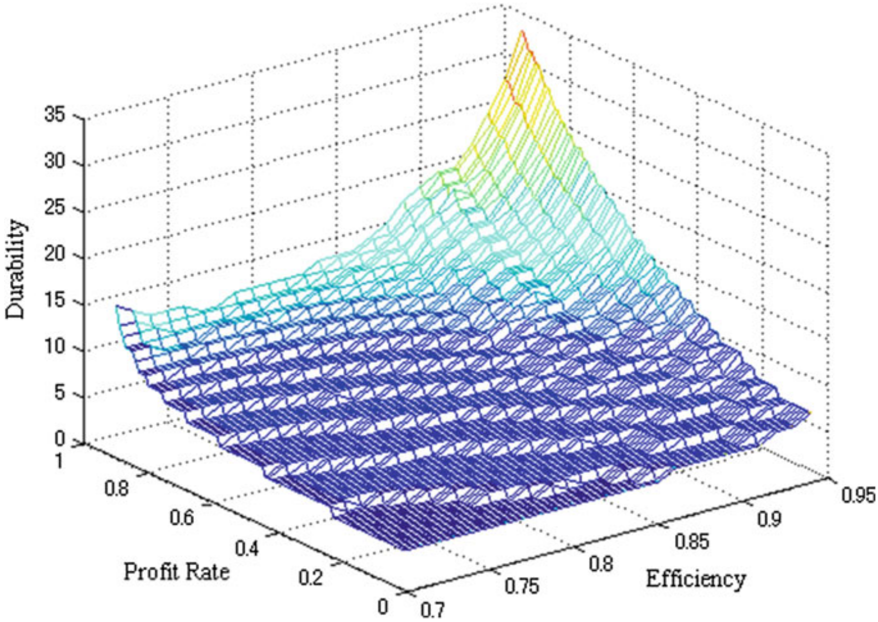


**Fig. 5.1** The economic determination of the durability of fixed capital: $\tau = \tau(\alpha, r)$

The figure demonstrates the *hardening effect*, which refers to the case where the efficiency of 1-year-old or 2-year-old aged fixed capitals is higher than that of brand-new goods. Thus, in this case, the output matrix $B$ can be expressed as follows.

$$B = \begin{pmatrix} 1 & \beta & \beta & 1 & 0.95\ldots & 0 & 0 & 0 & \ldots\ldots & 0 \\ 0.80 & & & & & 0.70 & & & & \\ & 0.80 & & & & & 0.70 & & & \\ & & 0.80 & & & & & 0.70 & & \\ & & & \ddots & & & & & \ddots & \\ & & & & \ddots & & & & & \ddots \\ 0 & 0 & 0 & \ldots\ldots & 0 & 1 & \beta & \beta & 1 & 0.95\ldots \end{pmatrix}, \qquad (5.4.2)$$

where $\beta > 1$ shows the efficiency of the 1-year-old and 2-year-old aged fixed capitals.

### 5.4.2 Production Prices and Activity Levels of the Production Process

Take the condition of fixed capital efficiency $\alpha$ as given in (5.4.1), and

$$\beta = 1.20, \; r = 0.20, \; n = 20, \qquad (5.4.3)$$

and we can solve the standard maximum problem (5.2.5) and its dual problem (5.2.6). The production price ratio $p$ is obtained as follows:

$$p = \big(0.3491\ 0.3576\ 0.2354\ 0.0889\ 0.0453\ 0.0148\ 0 \ldots 0\ 0.4630\big) \tag{5.4.4}$$
$$\propto \big(1\ 1.0242\ 0.6744\ 0.2546\ 0.1297\ 0.0423\ 0 \ldots 0\ 1.3262\big)\,. \tag{5.4.5}$$

Therefore, the endogenously determined durability of fixed capital is $\tau = 6$. The optimal solution $w_{max}$ is as follows:

$$w_{max} = pF = 0.4630. \tag{5.4.6}$$

Equation (5.4.5) shows the singularity of aged fixed capital, which is that the price of 1-year-old fixed capital has a higher evaluation value than the price of brand-new fixed capital.[1] The real wage rate $f^{**}$ in this case is as follows.

$$f^{**} = \frac{w_{max}}{p^0} = \frac{0.4630}{0.3491} = 1.3262. \tag{5.4.7}$$

On the other hand, the activity level $x$ of the production process is as follows:

$$x = {}^t(0, 0, 0, 0.0450, 0.1548, 0.1290, 0, \ldots, 0, 0.3668, 0.3056, 0.2547, 0.1608, 0, \ldots, 0) \tag{5.4.8}$$

Naturally, the optimal solution $u$ at this time is consistent with the value of a standard maximum problem, which is as follows:

$$u_{min} = Lx = 0.4630. \tag{5.4.9}$$

The output ratio $q = Bx$ can be calculated as follows:

$$q = {}^t\big(0.3081\ 0.2567\ 0.2140\ 0.1783\ 0.1486\ 1.238\ 0.1032\ 0 \ldots 0\ 1.2\big) \tag{5.4.10}$$
$$\propto {}^t\big(1\ 0.8333\ 0.6944\ 0.5787\ 0.4823\ 0.4019\ 0.3349\ 0 \ldots 0\ 3.8951\big). \tag{5.4.11}$$

Table 5.3 shows the result of simulation regarding the hardening effect and the endogenous determination of economic durability of fixed capital.

Figure 5.2 shows three dimensional relations among profit rate $r$, efficiency $\beta$ of aged fixed capital of 1- and 2-year-olds, and durability $\tau$.

[1] Assuming the profit rate $r = 0.20$ and the efficiency of 1-year-old and 2-year-old fixed capitals $\beta = 1.05, 1.10, 1.15, 1.20, 1.25$, the price of aged fixed capital of the 1-year-old $p^1$ is obtained through the above calculation as $p^1 = 0.9141, 0.9474, 0.9827, 1.0242, 1.0627$. It can be said that the evaluation value of the price of aged fixed capital of the 1-year-old rises in direct proportion to the value of $\beta$.

**Table 5.3** The endogenous determination of economic durability with the hardening effect

| $r$ | $\beta = 1.05$ | | $\beta = 1.10$ | | $\beta = 1.15$ | | $\beta = 1.20$ | | $\beta = 1.25$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $f^{**}$ | $\tau$ | $f^{**}$ | $\tau$ | $f^{**}$ | $\tau$ | $f^{**}$ | $\tau$ | $f^{**}$ | $\tau$ |
| 0.05 | 1.3536 | 6 | 1.3692 | 6 | 1.3479 | 5 | 1.3653 | 5 | 1.3853 | 4 |
| 0.10 | 1.3511 | 6 | 1.3572 | 6 | 1.3700 | 5 | 1.3525 | 5 | 1.3705 | 5 |
| 0.15 | 1.3289 | 7 | 1.3438 | 7 | 1.3606 | 6 | 1.3626 | 5 | 1.3566 | 5 |
| 0.20 | 1.3151 | 8 | 1.3304 | 7 | 1.3422 | 6 | 1.3262 | 6 | 1.3423 | 6 |
| 0.25 | 1.3007 | 8 | 1.3156 | 8 | 1.2974 | 7 | 1.3122 | 7 | 1.3279 | 6 |
| 0.30 | 1.2555 | 9 | 1.2688 | 8 | 1.2829 | 8 | 1.2974 | 7 | 1.3128 | 7 |
| 0.35 | 1.2399 | 10 | 1.2534 | 9 | 1.2674 | 9 | 1.2819 | 8 | 1.2968 | 7 |
| 0.40 | 1.2234 | 10 | 1.2370 | 10 | 1.2511 | 9 | 1.2655 | 9 | 1.2805 | 8 |
| 0.45 | 1.2062 | 11 | 1.2198 | 10 | 1.2339 | 10 | 1.2483 | 9 | 1.2632 | 9 |
| 0.50 | 1.1882 | 12 | 1.2019 | 11 | 1.2159 | 11 | 1.2303 | 10 | 1.2451 | 9 |
| 0.55 | 1.1697 | 12 | 1.1833 | 12 | 1.1972 | 11 | 1.2115 | 11 | 1.2262 | 10 |
| 0.60 | 1.1506 | 13 | 1.1641 | 12 | 1.1779 | 12 | 1.1921 | 11 | 1.2066 | 11 |
| 0.65 | 1.1311 | 14 | 1.1444 | 13 | 1.1581 | 13 | 1.1721 | 12 | 1.1864 | 12 |
| 0.70 | 1.1112 | 15 | 1.1244 | 14 | 1.1378 | 13 | 1.1516 | 13 | 1.1657 | 12 |
| 0.75 | 1.0911 | 15 | 1.1040 | 15 | 1.1172 | 14 | 1.1307 | 14 | 1.1446 | 13 |
| 0.80 | 1.0593 | 16 | 1.0600 | 16 | 1.0608 | 15 | 1.0614 | 15 | 1.0620 | 15 |
| 0.85 | 1.0399 | 17 | 1.0416 | 17 | 1.0431 | 16 | 1.0444 | 16 | 1.0457 | 15 |
| 0.90 | 1.0205 | 18 | 1.0229 | 17 | 1.0252 | 17 | 1.0273 | 17 | 1.0292 | 16 |
| 0.95 | 1.0009 | 18 | 1.0042 | 18 | 1.0072 | 18 | 1.0100 | 17 | 1.0125 | 17 |

## 5.5 Simulation with Physical Durability

When we solve the standard maximum problem (5.2.5) and its dual problem (5.2.6) under the above-mentioned parameters (5.3.4), durability becomes $\tau = 5$, the production price $p$ and the activity level of production process $x$ are determined by (5.3.5) and (5.3.9), respectively. Here, let $\bar{B}$ and $\bar{A}$ stand for the output- and input matrices of only operated processes, respectively.

$$\bar{B} = \begin{pmatrix} 1 & \dots & 1 & 1 & 0 & \dots & 0 & 0 \\ 0.80 & & & & 0.70 & & & \\ & \ddots & & & & \ddots & & \\ & & 0.80 & & & & 0.70 & \\ 0 & \dots & 0 & 0 & 1 & \dots & 1 & 1 \end{pmatrix}. \tag{5.5.1}$$

Let $L$ and $F$ stand for corresponding labour-input and wage-goods-bundle vectors, respectively. Under the conditions

$$\alpha = 1,\ r = 0.20,\ n = \tau = 5, \tag{5.5.2}$$

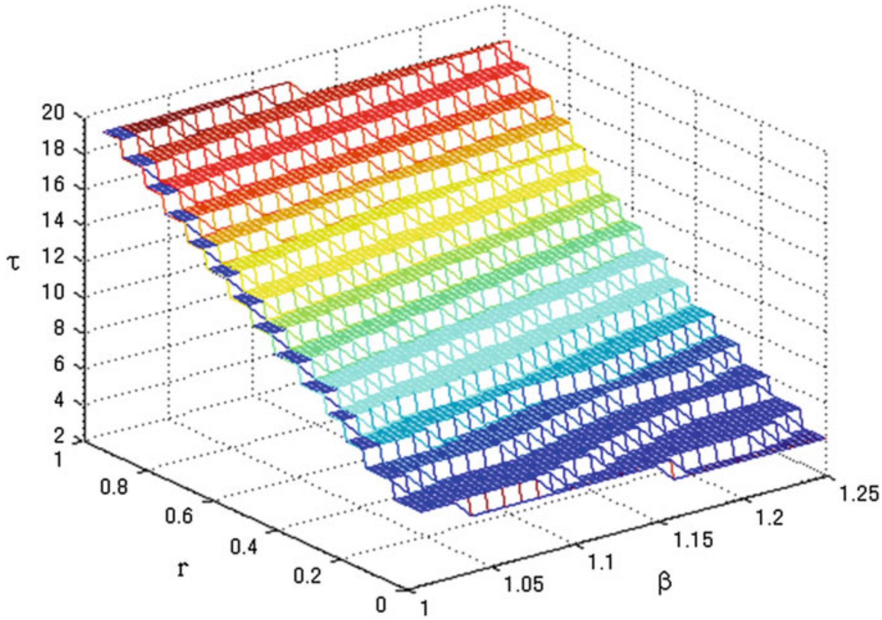


**Fig. 5.2** The economic determination of durability of fixed capital: $\tau = \tau(\beta, r)$

we can solve the standard maximisation problem (5.2.5) and its dual problem (5.2.6).[2] The production price $\bar{p}$ in the physical durability case is as follows:

$$\bar{p} = \begin{pmatrix}0.4096 & 0.3545 & 0.2885 & 0.2092 & 0.1141 & 0.5159\end{pmatrix} \tag{5.5.3}$$

$$\propto \begin{pmatrix}1 & 0.8656 & 0.7044 & 0.5109 & 0.2787 & 1.2596\end{pmatrix}. \tag{5.5.4}$$

Therefore, from the maximum wage rate $\bar{w}_{max} = \bar{p}\bar{F} = 0.5159$, the real wage rate $\bar{f}$ in this case is computed as follows:

$$\bar{f} = \frac{\bar{w}_{max}}{\bar{p}^0} = \frac{0.5159}{0.4096} = 1.2596. \tag{5.5.5}$$

On the other side, the activity level $\bar{x}$ of a production process becomes as follows:

$$\bar{x} = {}^t\begin{pmatrix}0.0506 & 0.3329 & 0 & 0 & 0 & 0.3987 & 0 & 0.3170 & 0.2642 & 0.2202\end{pmatrix}, \tag{5.5.6}$$

and the output ratio is computed from $\bar{q} = \bar{B}\bar{x}$.

$$\bar{q} = {}^t\begin{pmatrix}0.3835 & 0.3196 & 0.2663 & 0.2219 & 0.1849 & 1.2000\end{pmatrix} \tag{5.5.7}$$

$$\propto {}^t\begin{pmatrix}1 & 0.8333 & 0.6944 & 0.5787 & 0.4823 & 3.1295\end{pmatrix}. \tag{5.5.8}$$

[2] In this case the efficiency of fixed capital is constant, and the physical life span becomes $\tau$-year.

**Table 5.4** The determination of the real wage rate $\bar{f}$ in the physical durability case

| $r$ | $\alpha = 1$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\bar{f}$ | $\tau$ | $\bar{f}$ | $\tau$ | $\bar{f}$ | $\tau$ | $\bar{f}$ | $\tau$ | $\bar{f}$ | $\tau$ |
| 0.05 | 1.3030 | 5 | 1.2816 | 4 | 1.2458 | 3 | 1.2458 | 3 | 1.2458 | 3 |
| 0.10 | 1.3036 | 6 | 1.2675 | 4 | 1.2311 | 3 | 1.2311 | 3 | 1.2311 | 3 |
| 0.15 | 1.2890 | 6 | 1.2747 | 5 | 1.2529 | 4 | 1.2529 | 4 | 1.2161 | 3 |
| 0.20 | 1.2835 | 7 | 1.2596 | 5 | 1.2378 | 4 | 1.2378 | 4 | 1.2006 | 3 |
| 0.25 | 1.2738 | 8 | 1.2438 | 5 | 1.2438 | 5 | 1.2222 | 4 | 1.1848 | 3 |
| 0.30 | 1.2609 | 9 | 1.2411 | 6 | 1.2276 | 5 | 1.2061 | 4 | 1.2061 | 4 |
| 0.35 | 1.2453 | 10 | 1.2239 | 6 | 1.2108 | 5 | 1.1897 | 4 | 1.1897 | 4 |
| 0.40 | 1.2278 | 11 | 1.2144 | 7 | 1.2063 | 6 | 1.1936 | 5 | 1.1729 | 4 |
| 0.45 | 1.2088 | 12 | 1.2008 | 8 | 1.1882 | 6 | 1.1761 | 5 | 1.1761 | 5 |
| 0.50 | 1.1889 | 13 | 1.1815 | 8 | 1.1698 | 6 | 1.1698 | 6 | 1.1582 | 5 |
| 0.55 | 1.1687 | 15 | 1.1644 | 9 | 1.1577 | 7 | 1.1511 | 6 | 1.1399 | 5 |
| 0.60 | 1.1479 | 16 | 1.1457 | 10 | 1.1383 | 7 | 1.1320 | 6 | 1.1320 | 6 |
| 0.65 | 1.1270 | 18 | 1.1259 | 11 | 1.1219 | 8 | 1.1186 | 7 | 1.1128 | 6 |
| 0.70 | 1.1060 | 20 | 1.1051 | 11 | 1.1035 | 9 | 1.0987 | 7 | 1.0933 | 6 |
| 0.75 | 1.0850 | 22 | 1.0848 | 13 | 1.0829 | 9 | 1.0814 | 8 | 1.0786 | 7 |
| 0.80 | 1.0640 | 24 | 1.0639 | 14 | 1.0631 | 10 | 1.0623 | 9 | 1.0609 | 8 |
| 0.85 | 1.0430 | 26 | 1.0430 | 15 | 1.0426 | 11 | 1.0422 | 10 | 1.0416 | 9 |
| 0.90 | 1.0220 | 29 | 1.0220 | 17 | 1.0219 | 13 | 1.0217 | 11 | 1.0214 | 10 |
| 0.95 | 1.0010 | 33 | 1.0010 | 19 | 1.0010 | 14 | 1.0009 | 13 | 1.0009 | 13 |

Obviously, in the production process of fixed capital, the processes with 0-year-old and 1-year-old fixed capital are in operation, and in the process producing the consumption goods, the production processes with 0-year-old, 2-year-old, 3-year-old, and 4-year-old fixed capital are in operation.

We can solve the linear programming problem (5.2.5) under the conditions which regard durability $\tau$ in Table 5.2 as the physical lifespan of fixed capital and also fix efficiency $\alpha = 1$. Table 5.4 is obtained as a result of such a computation.

## 5.6 Concluding Remarks

### *5.6.1 On the Endogenous Determination of Economic Durability*

It is clear from Fig. 5.1 that economic durability of fixed capital is dependent on the profit rate and that durability tends to be prolonged, if possible, as the profit rate increases. This is our main result.

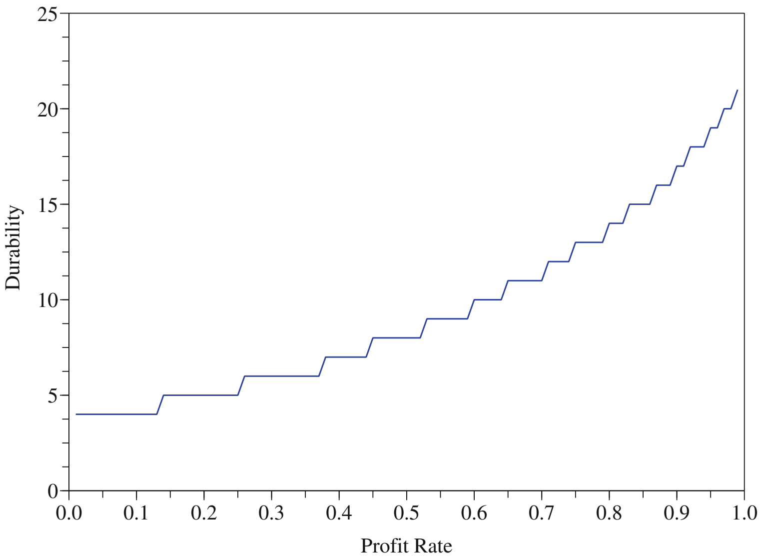


**Fig. 5.3** The relationship between the profit rate and economic durability ($\alpha = 0.90$)

In addition to the above, we can obtain some observations from our simulation. We can take a cross section of Fig. 5.1. Then, as to the relationship between the profit rate and economic durability, also as to the relationship between efficiency and economic durability, we can draw two-dimensional diagrams as follows.

(1) Fix $\alpha = \bar{\alpha}$. When the profit rate $r$ goes up, economic durability $\tau$ tends to become large. Figure 5.3 shows a sectional view of Fig. 5.1 in the case in which the real wage rate goes down.
(2) Fix $r = \bar{r}$. When $\alpha$ (the efficiency of fixed capital) falls, economic durability $\tau$ tends to become small. Figure 5.4 shows a sectional view of Fig. 5.1 in the case in which the real wage rate goes down.

### 5.6.2 *On the Hardening Effect of Fixed Capital*

From Table 5.3 and $\beta$, we can conclude as follows on the hardening effect of fixed capital.

(1) Fix the efficiency of 1-year-old and 2-year-old fixed capital as $\beta = \bar{\beta}$. When the profit rate $r$ rises and the real wage rate goes down, economic durability $\tau$ tends to become large.
(2) The real wage rate becomes higher in the case where the hardening effect is applicable as compared than otherwise.

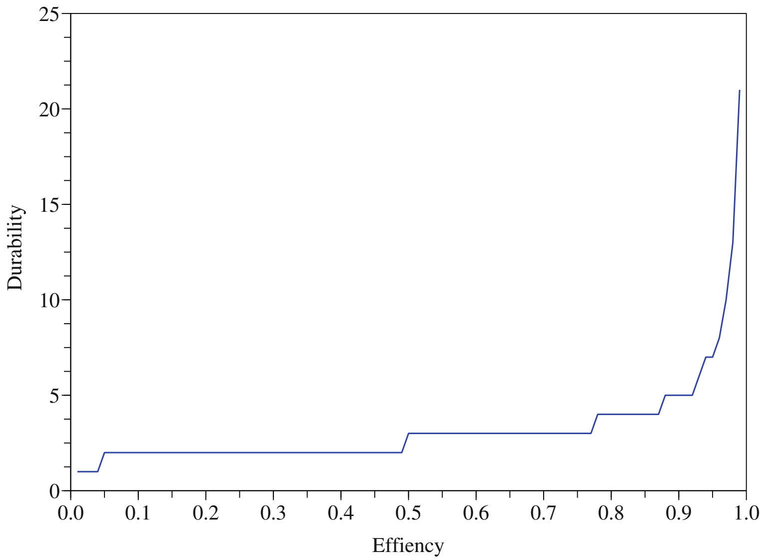


**Fig. 5.4** The relationship between the efficiency and economic durability ($r = 0.20$)

(3) The price of aged fixed capital is evaluated to be higher in the case where the hardening effect is applicable than otherwise. When $\beta$ exceeds a certain numerical value, it has been singularly found that the price evaluation of 1-year-old aged fixed capital is higher than that of brand-new or 0-year-old fixed capital.

### *5.6.3 Comparison with Physical Durability*

Figure 5.5 shows the wage-profit curve in the case of $\alpha = 0.90$ in Table 5.2 and the wage-profit curve in Table 5.4 by the same durability.

Furthermore, by the identical profit rate ($r = 0.20$), the price of aged fixed capital based on economic durability (5.3.6), the price of aged fixed capital based on the hardening effect (5.4.5), and the price of aged fixed capital with physical durability (5.5.4) are shown in Fig. 5.6. A comparative analysis of each item is as follows.

(1) $p$ is determined depending on both fixed capital efficiency $\alpha$ and profit rate $r$.
(2) $\bar{p}$ is determined only on the basis of the profit rate $r$.

In addition to the above, the price of aged fixed capital and the variation in the amount depreciated with physical durability are gentle as compared to economic durability.

Thus, by comparing the production price $p$ with $\bar{p}$, the difference between the two cases of depreciation, *i.e.*, in the case of economic durability and physical durability, will be made clear.

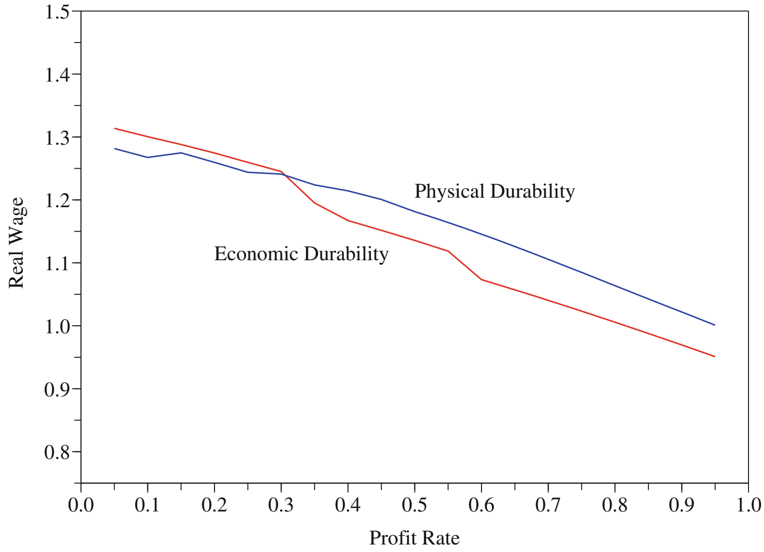


**Fig. 5.5** The wage-profit curve in an economic durability and a physical durability

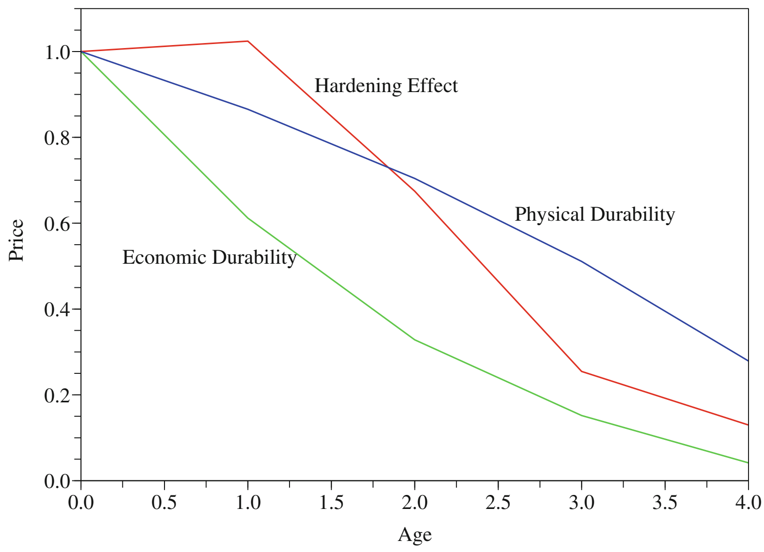


**Fig. 5.6** The aged fixed capital prices based on economic durability, physical durability, and hardening effect

The output ratios of aged fixed capital expressed by (5.3.12), (5.4.11), and (5.5.8) are mutually conformable under the same profit rate condition; this comparison is in contrast with the price ratio of fixed capital classified by ages. This simulation shows that the output ratio of aged fixed capital does not depend on efficiency, but it is dependent only on the profit rate.

### 5.6.4 Theoretical Implication

In the case of durability of fixed capital being given exogenously, prices and the wage rate are determined as functions of the profit rate. Besides, depreciation is determined as a function of the profit rate and given physical durability. By a linear programming method of this chapter, however, prices, the wage rate and economic durability are determined as functions of the profit rate completely, and consequently, depreciation is determined only as a function of the profit rate. Such an active role of the profit rate is vividly illustrated in the above.

# Chapter 6
# Marx-Sraffa Equilibria as Eigensystems

This chapter develops formal discussion of equilibrium of the Marx-Sraffa model from the angle of the eigenvalue and eigenvectors.

## 6.1 Mathematical Preliminaries

Some of mathematical framework will be explained in advance. This section is dedicated to the description of Moore-Penrose quasi inverses based on singular value decomposition.[1]

Take a system of homogeneous equations with a rectangular coefficient matrix of the form:

$$(A - \lambda B)x = 0. \tag{6.1.1}$$

where $A$ and $B$ are $m \times n$. The class of coefficient matrix of this form is called a *matrix pencil* of $A$ and $B$, and the problem here is to investigate if there exists a nontrivial solution $x$ satisfying (6.1.1). The set of values $\lambda$ such that $x \neq 0$ in (6.1.1) exists, is called the *spectrum* of the matrix pencil.

Suppose $\operatorname{rank}(A) = \operatorname{rank}(B) = n = \min(m, n)$. Premultiply the above by ${}^{t}\!A$, and it follows

$$({}^{t}\!AA - \lambda\, {}^{t}\!AB)x = 0.$$

From rank $A = n$, $\operatorname{rank}({}^{t}\!AA) = n$, and hence ${}^{t}\!AA$ is nonsingular. Therefore, one obtains

$$(I - \lambda({}^{t}\!AA)^{-1}\, {}^{t}\!AB)x = 0. \tag{6.1.2}$$

[1] As for details of mathematical description, refer to, *e.g.* Gantmacher (1959), Ben-Israel et al. (2013) and Strang (1976).

 
B. Li, *Linear Theory of Fixed Capital and China's Economy*,
DOI 10.1007/978-981-10-4065-8_6

Thus, the problem to find the spectrum of a matrix pencil $A - \lambda B$ is reduced to the eigenvalue problem of a square matrix $({}^t\!AA)^{-1}\,{}^t\!AB$.

The inversion procedure in the above can be expressed by means of Moore-Penrose inverse, which is represented *via* singular value decomposition (SVD) of matrices.

**Theorem 6.1.1** (SVD) *Suppose* $A(m \times n)$ *is of* $\operatorname{rank}(A) = r < \min(m, n)$. *Let* $\sigma_j(\neq 0),\ j = 1, \ldots, r$, *be singular values of* $A$, *and* $\Sigma = \begin{pmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_r \end{pmatrix}$. *Then, there exist orthogonal matrices* $U(m \times m)$ *and* $V(n \times n)$, *such that* $A = U \begin{pmatrix} \Sigma & O \\ O & O \end{pmatrix} {}^tV$.

The decomposition of $A$ in Theorem (SVD) is called the *singular value decomposition* of a matrix $A$.

Remark that the singular value decomposition of a matrix is unique.

For the above obtained singular value decomposition of matrices, a representation of MP-inverses of matrices is given by the following theorem.

**Theorem 6.1.2** (MP-inverse) *For a matrix* $A$ *in Theorem (SVD), an* $n \times m$ *matrix,* $X = V \begin{pmatrix} \Sigma^{-1} & O \\ O & O \end{pmatrix} {}^tU$ *satisfies the following four*: $AXA = A$, $XAX = X$, ${}^t(AX) = AX$, *and* ${}^t(XA) = XA$.

The matrix $X$ which satisfies the above four conditions is called the *Moore-Penrose inverse* of $A$, and denoted as $A^+$. Since the singular value decomposition of a matrix is unique, the Moore-Penrose inverse of a matrix is unique.

Remark that, by applying the notion of MP-inverse, one can write in (6.1.2):

$$({}^t\!AA)^{-1}\,{}^t\!A = A^+.$$

In the following discussion, the full rank matrices are dealt with, so that one should note:

**Lemma 6.1.1** *Suppose* $A$ *is of* $m \times n$ *and* $\operatorname{rank}(A) = \min(m, n)$. *Then,* $AA^+ = I$, *if* $m \leq n$; $A^+A = I$, *if* $m > n$.

(Proof is trivial, and hence omitted.)

Remark that if $A$ is nonsingular, then $A^+ = A^{-1}$.

**Lemma 6.1.2** *Let* $X$ *and* $Y$ *be of* $m \times n$ *and* $n \times m$ *matrices, respectively. If* $\operatorname{rank}(X) = \operatorname{rank}(Y) = \min(m, n)$, *eigenvalues of* $XY$ *are equal to those of* $YX$, *except zeros.*

A proof of the lemma is usually made *via* the notion of Schur's complement. In order to show the lemma, one has to make $U = \begin{pmatrix} W & X \\ Y & Z \end{pmatrix}$ and $V = \begin{pmatrix} W^{-1} & O \\ -YW^{-1} & I_m \end{pmatrix}$, where $W$ and $Z$ are of $m \times m$ and $n \times n$, respectively. Then, $VU = \begin{pmatrix} I & W^{-1}X \\ O & Z - YW^{-1}X \end{pmatrix}$, where $Z - YW^{-1}X$ is the Schur complement of $W$ in $U$. Therefore, $|U| = |W||Z - YW^{-1}X|$. Similarly, one obtains $|U| = |Z||W - XZ^{-1}Y|$. Now, let $W = \lambda I_m$ and $Z = \lambda I_n$. It follows that

$$\lambda^{m-n}|\lambda I_n - YX| = |\lambda I_m - XY|.$$

### 6.1.1 Marx-Sraffa Equilibria

In what follows, *non-basics are disregarded.*

**The starting problem**. As before, let $B$ and $M$ denote the $m \times n$ output- and the input matrix, respectively, where $m$ denotes the number of types of commodities, and $n$ indicates the number of processes. Durability of fixed capital is given.

Remark that the following theory does not require SON1–4.

The problem here is to find pairs of the profit factor $\alpha$ and corresponding nontrivial $p$ which fulfill the homogeneous equation

$$p(B - \alpha M) = \circ. \tag{6.1.3}$$

From the angle of formality, this is a problem to find the spectrum of a matrix pencil $B - \alpha M$.

**Definitions and lemmas**. Write SVDs of $B$ and $M$ as follows:

$$B = U \begin{pmatrix} \Sigma & O \end{pmatrix} {}^tV, \;\; M = S \begin{pmatrix} \Lambda & O \end{pmatrix} {}^tT,$$

where $U$ and $S$ are of $m \times m$, and $V$ and $T$ are of $n \times n$. Then, one obtains:

$$B^+ = V \begin{pmatrix} \Sigma^{-1} \\ O \end{pmatrix} {}^tU, \quad M^+ = T \begin{pmatrix} \Lambda^{-1} \\ O \end{pmatrix} {}^tS.$$

Marx-Sraffa price equilibrium is determined by a pair of $\alpha$ and corresponding $p \neq \circ$ that satisfies (6.1.3). That is, one must first define

$$\Omega = \{\alpha | \exists p \neq \circ \text{ such that } p(B - \alpha M) = \circ.\}$$

In case $\Omega \neq \emptyset$, one can define, for each $\alpha$,

$$\Omega_\alpha = \{p \neq \circ | p(B - \alpha M) = \circ.\}$$

On the other hand, in the case in which non-basics are disregarded, Marx-Sraffa activity equilibrium is defined by a pair of $\beta$ and $x$ fulfilling

$$(B - \beta M)x = 0. \tag{6.1.4}$$

Likewise as $\Omega$, one can define:

$$\Xi = \{\beta | \exists x \neq 0 \text{ such that } (B - \beta M)x = 0\},$$

and corresponding

$$\Xi_\beta = \{x \neq 0 | (B - \beta M)x = 0\}$$

for each $\beta$.

In a similar manner, one can define, for matrix pencils $I - \alpha M B^+$ and $I - \beta B^+ M$, the following:

$$\begin{aligned}
\Omega^* &= \{\alpha | \exists p \neq \circ \text{ such that } \; p(I - \alpha M B^+) = \circ\}, \\
\Omega^*_\alpha &= \{p \neq \circ | p(I - \alpha M B^+) = \circ\}, \\
\Xi^* &= \{\beta | \exists x \neq 0 \text{ such that } (I - \beta B^+ M)x = 0\}, \\
\Xi^*_\beta &= \{x \neq 0 | (I - \beta B^+ M)x = 0\}.
\end{aligned}$$

The rank condition is assumed in the subsequent discussion:

RC $\quad \mathrm{rank}(B) = m$.

Remark that this condition implies $m \leq n$.

From Lemma 6.1.1, it follows:

**Lemma 6.1.3** *Assume* RC. *(i)* $\Omega \times \Omega_\alpha \subseteq \Omega^* \times \Omega^*_\alpha$. *(ii)* $\Xi^* \times \Xi^*_\beta \subseteq \Xi \times \Xi_\beta$.

**Price equilibria as eigensystems**. In lieu of SVD, more loose decomposition of matrices is applied here. Consider a nonsingular decomposition of $B$ such as

$$B = X(B_1, O)Y, \tag{6.1.5}$$

where $X$, $B_1$ and $Y$ are nonsingular matrices.[2] Then, from mathematical manipulation it follows that $p(I - \alpha M B^+) = \circ$ implies $p(B - \alpha M)Y^{-1}\hat{I}_m = \circ$.

Take $Y = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix}$, and $Y^{-1} = \begin{pmatrix} \bar{\Upsilon}_{11} & * \\ W & * \end{pmatrix}$, where $W = -(Y_{22} - Y_{21}Y_{11}^{-1}Y_{12})^{-1}$ $Y_{21}Y_{11}^{-1}$.

Since $Y_{21}$ can be taken arbitrary, take $Y$ with $Y_{21} = O$, and $u^1 \Upsilon_{11} = \circ$. Since $\bar{\Upsilon}_{11}$ is nonsingular, $u^1 = \circ$.

Next, take $Y$ in such a way that $W$ is an arbitrary $(0, 1)$-matrix with $1 \leq \mathrm{rank}(W) \leq \mathrm{rank}(Y_{21}) = \min(m, n - m)$, so that $u^2 W = \circ$ implies $u^2 = \circ$. Therefore, $u = p(B - \alpha M) = \circ$.

---

[2] Consider an $m \times n$ matrix $G = (G_1, G_2)$ with $\mathrm{rank}(G) = m$, where $G_1$ and $G_2$ are of $m \times m$ and of $m \times (n - m)$ respectively. Then, $G$ can be decomposed as $G = X(\Gamma, O)Y$, where $\Gamma$ is an $m \times m$ nonsingular matrix, and $X$ and $Y$ are $m$- and $n$-dimensional nonsingular matrices respectively. It is evident that triplets of $X$, $\Gamma$ and $Y$ are chosen from infinite possible combinations. It should be noted that the choice of them is under some restrictions. Let $Y = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix}$, and one has $Y_{11} = (X\Gamma)^{-1}G_1$ and $Y_{12} = (X\Gamma)^{-1}G_2$. This means that the upper partitions of $Y$ are restricted, whilst the lower partitions, in particular $Y_{21}$, are arbitrary, insofar as $Y$ is nonsingular.

Hence, the first main theorem follows:

**Theorem 6.1.3** *Assume* RC. *Then,* $\Omega^* \times \Omega_\alpha^* = \Omega \times \Omega_\alpha$.

Therefore, determination of the price equilibrium is transformed into the equivalent eigensystem with mostly RC alone.

**Another eigensystem of prices**. Beside $p = \alpha p M B^+$ in the above, price equilibrium can be based on another equation: $pBM^+ = \alpha p$. The following lemma makes clear the relationship between the eigenvalues of $BM^+$ and $MB^+$.

**Lemma 6.1.4** *Assume* RC. *Nonzero eigenvalues of $MB^+$ are reciprocals of those of $BM^+$.*

Since $\mathrm{rank}(MB^+) = m$, so that $(MB^+)^+ = (MB^+)^{-1}$, and one has only to show $(MB^+)^+ = BM^+$. In fact,

$$MB^+ = S \begin{pmatrix} \Lambda & O \end{pmatrix} {}^tTV \begin{pmatrix} \Sigma^{-1} \\ O \end{pmatrix} {}^tU, \quad BM^+ = U \begin{pmatrix} \Sigma & O \end{pmatrix} {}^tVT \begin{pmatrix} \Lambda^{-1} \\ O \end{pmatrix} {}^tS.$$

These fulfill the four conditions for MP-inverses, and it follows that they are MP-inverses of each other. Hence, the lemma holds.

**Profits and growth**. The next problem is to confirm that growth factors are equal to profit factors, in case that non-basics are ignored. We consider here the relationship between the profit factor $\alpha$ and the growth factor $\beta$.

As opposed to price equilibria, some difficulties will be encountered in the activity equilibrium case.

As seen in the above, $\Omega$ and $\Omega^*$ are based on matrices with $m$ rows, so that their ranks are easily dealt with. When the activity equilibria are taken up, however, the square matrix $B^+M$ is of $n \times n$, whilst $\mathrm{rank}(B^+M) = m \leq n$. Since $B^+M$ possess zero eigenvalues, some of $\beta$s in $\Xi^*$ are not bounded.[3] This possibility of unbounded growth factors comes purely out of mathematical manipulation. With regard to this, some detailed points on the removal of unbounded growth factors are taken up here. $\Xi^*$ is the linkage between price and activity equilibria, nonetheless.

Define

$$\Xi^\dagger \equiv \{\beta \in \Xi^* | \beta < \infty\},$$

and in view of Lemma 6.1.2, one obtains:

**Lemma 6.1.5** $\Xi^\dagger = \Omega^*$.

[3]Remark that $\beta$s are reciprocals of eigenvalues of $B^+M$. In arguments of economics, unbounded growth factors are usually disregarded.

From the above, it follows that $\Omega = \Omega^* = \Xi^\dagger \subseteq \Xi$. The remaining task is to introduce some restrictions to establish full equalities in the right-most part.

As for a relationship between $\Omega$ and $\Xi$, one should remark that the rank of the coefficient matrices are evaluated in a similar manner by the greatest dimensions of nonzero minors of the matrices: $\mathrm{rank}(B - \alpha M) = \mathrm{rank}\ {}^t(B - \alpha M)$. However, those matrices are transposes of each other, so that, the range of spectra differs. In case $m \le n$, (6.1.4) has nontrivial solutions, even if $\mathrm{rank}(B - \beta M) = m$, whilst, (6.1.3) possesses trivial solutions alone if $\mathrm{rank}(B - \alpha M) = m$. That is, if $m \le n$, then there exists a $\beta \in \Xi$ such that $\mathrm{rank}(B - \beta M) = m$, whilst $\Omega \ne \emptyset$ includes only those $\alpha$s such that $\mathrm{rank}(B - \alpha M) \le m - 1$. Hence, it is appropriate to exclude such $\beta$s with $\mathrm{rank}(B - \beta M) = m$ from $\Xi$: one should define

$$\tilde{\Xi} = \{\beta \in \Xi \,|\, \mathrm{rank}\,(B - \beta M) < m\}.$$

Note that $\Omega^*$ does not contain $\alpha$s with $\mathrm{rank}(I - \alpha M B^+) = m$, and one obtains:

**Lemma 6.1.6** $\Omega^* \subseteq \tilde{\Xi}$.

Since spectra for $B - \alpha M$ and ${}^t(B - \alpha M)$ with $\mathrm{rank}(B - \alpha M) \le m - 1$ coincide with each other, one obtains the following:

**Lemma 6.1.7** $\Omega = \tilde{\Xi}$.

From Theorem 6.1.3 $\Omega = \Omega^*$, and from Lemma 6.1.3, $\Xi^* \subseteq \Xi$, whilst, from Lemma 6.1.5, $\Omega^* = \Xi^\dagger$. From Lemma 6.1.7, one obtains $\Omega = \tilde{\Xi}$. Therefore, one obtains the second main theorem:

**Theorem 6.1.4** *Assume* RC. $\Omega = \Omega^* = \tilde{\Xi} = \Xi^\dagger$.

Thus, the sets of the profit and unbounded growth factors are all equal.

**Remarks on activity levels and overall**. In the above, the restriction with respect to ranks are considered as $\iota = \mathrm{rank}(B - \alpha M) \le m - 1$. Then, the number of elementary solutions of the homogeneous equations $(B - \beta M)x = 0$ is $n - \iota$. Hence, there exist for a particular $\beta \in \tilde{\Xi}$ at most $n - \iota$ nontrivial solutions $x \ne 0$ for $(B - \beta M)x = 0$. In the meantime, the number of linearly independent eigenvectors $x \in \tilde{\Xi}_\beta$ corresponding to that $\beta$ does not exceed the multiplicity of $\beta$ as an eigen root of the characteristic equation of $B^+ M$.

Note that a similar lemma like Lemma 6.1.4 holds with respect to $B^+ M$ and $M^+ B$.

It should be remarked, however, that the square matrices, like $M B^+$ and $B M^+$, are not necessarily nonnegative matrices, and hence one cannot assert *a priori* that there exist nontrivial solutions such as $p \geq \circ$ or $x \geq 0$. Existence of such a solution should be ensured by another context.

## 6.2 Possibility of Unstable Marx-Sraffa Equilibrium

In the previous section, equations that define Marx-Sraffa equilibria of production prices and activity levels are transformed to equivalent eigenvalue problems. In this section, numerical examples to compute eigenvalues and eigenvectors corresponding to them will be given.

### *6.2.1 A Basis of Numerical Computation*

We summarize the eigensystem of Marx-Sraffa equilibria as follows:

$$p = \alpha p M B^+,$$
$$x = \beta B^+ M x.$$

We compute eigenvalues of $C = MB^+$ and $D = B^+M$ and eigenvectors corresponding to them. We check the results expressed in Lemmas.

Although the results obtained in the preceding section does not depend on a specific rule to generate processes, the generating rule of processes equipped with fixed capital will be taken into account in this section.

As shown in the above, the computation here is based on $B^+$, and not on $M^+$. This is due to a computational reason. Generally speaking, $M$ has more zero entries than $B$, which makes it difficult to compute $M^+$ numerically.

As for numerical settings, input coefficients of fixed capital of type 1 and 2, current goods, and labour input are set as in Table 6.1.

From the above, the input coefficient matrix $A = (a_{ij})$, the output coefficient matrix $B = (b_{ij})$, and the labour input vector $L = (l_j)$ are given. The wage goods bundle $F$ is given as follows:

$$F = {}^t\big(0\ 0\ 0\ 0\ 0\ 0\ 0.6\ 0.8\big)$$

Hence, one obtains: $M = A + FL$.

**Table 6.1** Numerical settings of the Marx-Sraffa model

| | | | | |
|---|---|---|---|---|
| $k_{11} = 0.40$ | $k_{12} = 0.50$ | $k_{13} = 0.35$ | $k_{14} = 0.45$ | $k_{15} = 0.60$ |
| $k_{21} = 0.30$ | $k_{22} = 0.25$ | $k_{23} = 0.50$ | $k_{24} = 0.40$ | $k_{25} = 0.55$ |
| $a_{11} = 0.25$ | $a_{12} = 0.40$ | $a_{13} = 0.20$ | $a_{14} = 0.30$ | $a_{15} = 0.50$ |
| $l_1 = 0.10$ | $l_2 = 0.20$ | $l_3 = 0.15$ | $l_4 = 0.10$ | $l_5 = 0.25$ |

### 6.2.2 Marx-Sraffa Price Equilibrium

The characteristic equation of $C = MB^+$ is

$$\begin{aligned}\lambda^8 + 1.54\lambda^7 + 0.645\lambda^6 - 0.618\lambda^5 - 1.376\lambda^4 \\ - 0.292\lambda^3 + 0.007\lambda^2 + 0.001\lambda = 0.\end{aligned} \tag{6.2.1}$$

Eigenvalues of $C$ are as follows:

$$\begin{aligned}\lambda_1 = 0.893, \lambda_2 = 0.060, \lambda_3 = -0.051, \lambda_4 = -0.255, \lambda_5 = -1.220, \\ \lambda_6,\ \lambda_7 = -0.484 \pm 0.933i, \lambda_8 = 0.\end{aligned}$$

The eigenvector corresponding to the maximum positive real eigenvalue $\lambda_1$ of $C$ is shown as follows:

$$p^1 = \left(0.296\ 0.156\ 0.437\ 0.308\ 0.162\ 0.342\ 0.342\ 0.585\right).$$

Also, $\tilde{p}^1$ corresponding to $\lambda_1$, the first element of which is 1, becomes:

$$\tilde{p}^1 = \left(1\ 0.528\ 1.478\ 1.040\ 0.549\ 1.156\ 1.156\ 1.978\right) \propto p^1. \tag{6.2.2}$$

The uniform profit rate is calculated as follows:

$$r = {\lambda_1}^{-1} - 1 = 0.12008. \tag{6.2.3}$$

$p^1$ corresponding to $\lambda_1$ gives the equilibrium production price ratio.

$C$ is not a nonnegative matrix. However, there exists a positive eigenvalue and a positive left eigenvector corresponding to it.

The first five elements of $p^1$ give the production prices of fixed capital. It can be confirmed, as seen in the following, that they also represent the prices based on the depreciation rate, which depends on the profit rate.

The depreciation rates based on the number of years $\tau$, in which fixed capital will be operated, $\varphi(r, \tau) = \dfrac{1}{\sum_{s=0}^{\tau-1}(1+r)^s}$, are computed as follows:

$$\varphi(r, 3) = 0.296,\ \ \varphi(r, 2) = 0.472.$$

The production prices of aged fixed capital, based on the depreciation rate, are as follows[4]:

$$\frac{p_1^1}{p_1^0} = \frac{p_2^2}{p_2^1} = 1 - \varphi(r, 2) = 0.528, \ \frac{p_2^1}{p_2^0} = 1 - \varphi(r, 3) = 0.704.$$

On the other hand, the equilibrium ratios of production price $p^1$ corresponding to $\lambda_1$ satisfy the following:

$$\begin{gathered}\left(p^1{}_1 \ p^1{}_2\right) \propto \left(1 \ 0.528\right),\\ \left(p^1{}_3 \ p^1{}_4 \ p^1{}_5\right) \propto \left(1 \ 0.704 \ 0.372\right).\end{gathered}$$

The above calculation shows the following: if the depreciation rate is defined as $\varphi(r, \tau)$, the equilibrium ratio of the production price of the aged fixed capital obtained as an eigenvector of $C$ and the production price ratio of the aged fixed capital obtained by means of the depreciation rate are the same.

### *6.2.3 Marx-Sraffa Activity Equilibrium*

The characteristic equation of $D = B^+M$ is given as follows:

$$\begin{aligned}\lambda^{22}(\lambda^8 + 1.54\lambda^7 + 0.645\lambda^6 - 0.618\lambda^5 - 1.376\lambda^4 \\ -0.292\lambda^3 + 0.007\lambda^2 + 0.001\lambda) = 0.\end{aligned} \tag{6.2.4}$$

The eigenvalues of $D$ are obtained by the set of eigenvalues $\lambda_1, \ldots, \lambda_8$ of $C$, added by duplicate roots 0s of (6.2.4).

The right eigenvector $z^1$ of $C$ corresponding to $\lambda_1$ is shown below:

$$\begin{aligned} z^1 = {}^t(&0.236, 0.207, 0.209, 0.221, 0.222, 0.194, 0.161, 0.131, 0.139, 0.143,\\ &0.150, 0.120, 0.309, 0.272, 0.265, 0.296, 0.286, 0.252, 0.113, 0.078,\\ &0.078, 0.096, 0.095, 0.061, 0.152, 0.104, 0.103, 0.129, 0.126, 0.081).\end{aligned}$$

$D$ is not always a nonnegative matrix, but there exist a positive eigenvalue $\lambda_1$ and a positive right eigenvector $z^1$ corresponding to it. The set of $\lambda_1$ and $z^1$ determine the steady state of the Marx-Sraffa model on the activity level side.

The uniform growth rate $g$ is equal to the uniform profit rate $r$, *i.e.* $g = r$.

---

[4] $\frac{p_2^2}{p_2^0} = (1 - \varphi(r, 2))(1 - \varphi(r, 3)) = 0.528 \times 0.704 = 0.372$. For details see Kurz and Salvadori (1995).

### *6.2.4 The Equilibrium Quantity of the Marx-Sraffa Model*

The ratios of the equilibrium quantities of the quantity system are given by $Bz^1$. That is,

$$q^1 \equiv Bz^1 = \begin{pmatrix} 1.288 \\ 1.150 \\ 0.844 \\ 0.754 \\ 0.673 \\ 1.679 \\ 0.521 \\ 0.695 \end{pmatrix} \propto \tilde{q}^1 \equiv \begin{pmatrix} 1 \\ 0.893 \\ 0.656 \\ 0.585 \\ 0.523 \\ 1.304 \\ 0.405 \\ 0.539 \end{pmatrix}. \tag{6.2.5}$$

The first two elements of the quantity ratio (the quantity ratios of the brandnew and aged fixed capital of type 1) and the third to fifth elements (the quantity ratio of the brandnew and aged fixed capital of type 2) constitute a geometrical progression, which is increasing by the ratio ('the growth rate $+1$'). That is,

$$\frac{{q^1}_1}{{q^1}_2} = \frac{{q^1}_3}{{q^1}_4} = \frac{{q^1}_4}{{q^1}_5} = \frac{1}{\lambda_1} = 1.12008. \tag{6.2.6}$$

This implies that the output quantity ratio of fixed capitals, increases by the uniform growth rate.

### *6.2.5 The Dynamics of the Production Price System*

The basic equation of the dynamic Marx-Sraffa production price system is represented as follows:

$$p(t+1)B = (1+r)p(t)M. \tag{6.2.7}$$

This is expressed as

$$p(t+1) = (1+r)p(t)MB^+. \tag{6.2.8}$$

Since the set of eigenvalues and left eigenvectors of $C$ are already known, these values can be used to obtain a time series of the production price.

As shown by the aforementioned numerical simulations, the positive eigenvalue $\lambda_1$ of $C$ is not the largest eigenvalue in absolute values, so $\lambda_1$ and $p^1$, a pair of which determines the equilibrium, cannot dominate the dynamic process. Unless an initial value is strictly equal or proportional to $p^1$, a negative price ratio will emerge sooner or later. Take an initial value as follows:

$$p(0) = \begin{pmatrix} 1 & 0.5 & 1.478 & 0.985 & 0.493 & 1.156 & 1.156 & 1.978 \end{pmatrix}, \; r = \frac{1}{\lambda_1} - 1.$$

**Table 6.2** Time series of production prices

| $t$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $p_1^o$ | 0.968 | 0.995 | 0.987 | 0.993 | 0.951 | 1.053 | 0.912 | 1.044 | 0.932 | 1.095 |
| $p_1^1$ | 0.560 | 0.457 | 0.603 | 0.430 | 0.630 | 0.360 | 0.776 | 0.152 | 0.998 | **−0.075** |
| $p_2^o$ | 1.446 | 1.462 | 1.450 | 1.485 | 1.406 | 1.520 | 1.399 | 1.517 | 1.360 | 1.637 |
| $p_2^1$ | 1.103 | 1.002 | 0.945 | 1.142 | 1.016 | 0.866 | 1.222 | 1.019 | 0.780 | 1.274 |
| $p_2^2$ | 0.552 | 0.618 | 0.430 | 0.577 | 0.633 | 0.429 | 0.490 | 0.820 | 0.222 | 0.625 |
| $p_3$ | 1.113 | 1.149 | 1.162 | 1.117 | 1.120 | 1.219 | 1.053 | 1.169 | 1.172 | 1.146 |
| $p_4$ | 1.116 | 1.151 | 1.146 | 1.140 | 1.102 | 1.222 | 1.049 | 1.201 | 1.098 | 1.242 |
| $p_5$ | 1.924 | 1.958 | 1.964 | 1.953 | 1.899 | 2.051 | 1.853 | 2.005 | 1.914 | 2.077 |

The iteration is made upto the 10-th period, and we get the above Table 6.2.[5]

As shown in Table 6.2, in the 10-th period, $p_1^1$ becomes negative.

### 6.2.6 *The Dynamics of the Activity and the Quantity Systems*

The basic equation of the dynamic Marx-Sraffa activity system is expressed by:

$$Bx(t) = Mx(t+1). \tag{6.2.9}$$

This is transformed into the normal form as follows:

$$x(t+1) = M^{+}Bx(t). \tag{6.2.10}$$

Since the positive real eigenvalue of $M^{+}B$ corresponding to the nonnegative eigenvector is not the largest in absolute values, it cannot dominate the dynamic process. Thus, starting from any initial value, a negative activity level or a negative quantity will emerge sooner or later.

[5] In setting up the initial value, the price ratio portion of the fixed capital is calculated by the straight-line depreciation method, and the price ratios of brand new fixed capital of types 1 and 2 are taken as $\frac{p^1{}_3}{p^1{}_1}$, with an equilibrium ratio of $p^1$. On the other hand, the price ratios of current goods and consumption goods 1 and 2 are taken as $\frac{p^1{}_6}{p^1{}_1}$, $\frac{p^1{}_7}{p^1{}_1}$, and $\frac{p^1{}_8}{p^1{}_1}$, respectively, the equilibrium ratio being $p^1$. That is, the initial value is given as follows:

$$p(0) = \left(1 \;\; \tfrac{1}{2} \;\; 1\cdot\frac{p^1{}_3}{p^1{}_1} \;\; \tfrac{2}{3}\cdot\frac{p^1{}_3}{p^1{}_1} \;\; \tfrac{1}{3}\cdot\frac{p^1{}_3}{p^1{}_1} \;\; \frac{p^1{}_6}{p^1{}_1} \;\; \frac{p^1{}_7}{p^1{}_1} \;\; \frac{p^1{}_8}{p^1{}_1}\right).$$

**Table 6.3** Time series of quantities

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $q_1^o$ | 1.288 | 1.210 | **−0.541** | **−9.051** | **−176.578** | ... |
| $q_1^1$ | 1.150 | 1.288 | 1.210 | **−0.541** | **−9.051** | ... |
| $q_2^o$ | 0.844 | 1.056 | **−0.460** | 17.260 | **−347.093** | ... |
| $q_2^1$ | 0.754 | 0.844 | 1.056 | **−0.460** | 17.260 | ... |
| $q_2^2$ | 0.673 | 0.754 | 0.844 | 1.056 | **−0.460** | ... |
| $q_3$ | 1.679 | 1.934 | 1.366 | 19.350 | **−301.242** | ... |
| $q_4$ | 0.521 | 0.753 | 1.000 | **−28.220** | 276.926 | ... |
| $q_5$ | 0.695 | 0.684 | 3.360 | 2.642 | 359.006 | ... |

Now, assuming that the initial value $x(0)$ of the activity level is again an approximate of equilibrium ratio $z^1$, we repeat the iteration of (6.2.10) and trace the time series of the quantity $q(t) = Bx(t)$. The Table 6.3 shows the results.

The above simulations exemplify instability of dynamics of both production prices and quantities of the Marx-Sraffa model.

## 6.3 Concluding Remarks

So far, we developed a way to construct the eigenvalue problem which is equivalent to finding equilibrium of Marx-Sraffa models defined in terms of equalities, which include aged fixed capital. The MP-inverse approach, as a direct method, was introduced as a major tool of analysis. This method can be applied to a general joint production case.

Result of the direct method indicates that if aged fixed capital is treated as flow, then production price equilibrium will not be stable.

Although the eigensystem from Okishio-Nakatani's reduction is not operational for computing equilibria, it suggests that relative ratios of equilibrium production prices are still stable. This implies that if the depreciation is taken into account to determine aged fixed capital exogenously, then production price equilibrium of brandnew commodities is stabilised. That is, the accounting system of depreciation is a mechanism to stabilize the formation of production prices of commodities.

# Chapter 7
# Fixed Capital and China's Economy 1995–2000

## 7.1 Introduction

In this chapter, we aim at investigating growth and distribution of China's economy on the basis of the input-output tables and fixed capital data. The tentative goal is to draw the wage-profit curves of China's economy, and obtain an overview of the position of the actual situation of growth and distribution of China's economy. We carry out this investigation focussing on fixed capital. Since the available data on fixed capital are very limited, it is necessary to start the task from estimating the fixed capital coefficients.

On the basis of the standard system of Sraffa (1960), Fujimori (1992a) developed a novel and original method to estimate the marginal fixed capital coefficient by employing the Japan's gross investment matrix data of fixed capital. The first part of this chapter will be spent for an outline of the Sraffa-Fujimori method. That is to say, the method employed in this chapter reallocate all of *consumption* items of the *final demand* to investment items proportionally and computed, in the state of the zero consumption, the marginal fixed capital coefficient, so that it is the computation of the standard system of Sraffa. Indeed, from the angle of computation, to evaluate the state in which zero consumption is observed is equivalent to either to look for the standard system of the original system concerned, or to evaluate just the potential greatest growth rate. We adhere fundamentally to Fujimori (1992b), with minor corrections and improvements, though.

Second, we apply this method to input-output tables of China's economy, and estimate the marginal fixed capital coefficients for 1987–2000.

Third, we draw the wage-profit curves of China's economy in a von Neumann-Leontief framework, by applying fixed capital coefficients estimated as above.[1, 2]

[1] As for Sraffa's joint-production system with fixed capital and the Leontief system with brandnew fixed capital, see Fujimori (1982) and Li and Fujimori (2013).

[2] The analysis frameworks on linear programming of the von Neumann, Marx, Leontief models are pioneered by Morishima (1964, 1973), Fujimoto (1975) and Fujimori (1982, 1992b). Especially, on the premise to define the depreciation rate by the pension method,



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Fourth, we discuss the short- and long-run features of China's economy of the period by comparing the short- and long-run wage-profit curves, where the short run indicates the case in which fixed capital is ignored.

## 7.2 Estimation of Marginal Fixed Capital Coefficients

### 7.2.1 Basic Framework

Okishio and Nakatani (1975) reduced a joint production system *à la* Marx-Sraffa with aged fixed capital alone as joint-products to a Leontief production system that consists of only brand-new goods, called SON. (See Chap. 2.)

The framework of SON is as follows. Let $A, K, F, L$ stand for input coefficient matrix, fixed capital input coefficient matrix, bundle of wage goods, labour input vector, and let $p$ and $r$ be the production price vector of only brand-new goods, and *uniform profit rate*, respectively.

$$p = pM(r), \tag{7.2.1}$$
$$M(r) = (\widehat{\psi}(r) + rI)K + (1 + r)(A + FL). \tag{7.2.2}$$

$\widehat{\psi}(r)$ is a diagonal matrix with the *rate of depreciation* $\psi_i(r)$ in the diagonal, where for *durability* $\tau_i$ of fixed capital $i$,

$$\psi_i(r) = \left( \sum_{h=0}^{\tau_i - 1} (1 + r)^h \right)^{-1}. \tag{7.2.3}$$

Remark that in the above formulation, one should assume physical durability of fixed capital with constant efficiency.[3]

If non-productive consumption is disregarded, the Marx-Sraffa activity level system can be similar to the Leontief output system with extra brand-new fixed capital.

(Footnote 2 continued)
Fujimori (1992b) expanded the joint production model of Morishima (1964) and Fujimoto (1975), and made an abridgement of the linear programming model only consisting of brand-new goods by a kind of rational operation. In addition, for the other empirical contributions, refer to *e.g.* Han and Schefold (2006).

[3]Remark that

$$\psi_i(r) = \left( \sum_{h=0}^{\tau_i - 1} (1 + r)^h \right)^{-1} = \frac{r}{(1 + r)^{\tau_i} - 1},$$

where $\tau_i$ stands for durability of the type $i$ of fixed capital. Equation (7.2.3) is the definition of depreciation rate of brand-new (0-age) fixed capital.

The equilibrium output system corresponding to the equilibrium production price system (7.2.1) is given by the following:[4]

$$q = M(g)q, \tag{7.2.4}$$

where $q$ and $g$ are an output vector of only brand-new goods and the *uniform growth rate*, respectively, with $g = r$.

Obviously, for an $r \geq 0$, $M(r)$ is a non-negative matrix, so that it has the Perron-Frobenius eigenvalue 1, and the nonnegative left (right) eigenvector corresponding to 1.

Now, the data of $K$ is not available; only the data of the gross investment matrix may be available as fixed capital data in the actual input-output tables. Therefore, the next subsection will illustrate how the fixed capital coefficient matrix $K$ in the above-mentioned theoretical model is estimated from the gross investment data of fixed capital.

### *7.2.2 Sraffa-Fujimori Method*

The intermediate input $X_{ij}$, final demand $Y_i$, and total output $X_i$ of an input-output table fulfill the following relations.

$$X_i = \sum_{j=1}^{n} X_{ij} + Y_i.$$

The input coefficient matrix $A = (a_{ij})$ is given by

$$a_{ij} = \frac{X_{ij}}{X_j}.$$

Let $x$, $I$ and $C$ stand for the output-, the investment- and the consumption-vector, respectively, and

$$x = Ax + I + C \tag{7.2.5}$$

will be obtained from the input-output table. $I$ is the sum total of inventory investment and the gross investment of fixed capital.

Hereafter, we try to find the growth rate of Sraffa's standard system obtained by the following simulation, in which $C$ is equally assigned to $I$ in the final-demand item. It is necessary to consider investment of both nondurable capital goods and fixed capital.

[4]For a detailed discussion, which includes fixed capital, see *e.g.* Fujimori (1982), Schefold (1989, Part II) and Kurz and Salvadori (1995, Chap. 7).

Since inventory investment can be regarded as the accumulation of nondurable goods, it is set to $gAx$, where $g$ denotes the *uniform growth rate*.

On the other hand, since the gross investment of fixed capital is divided into net investment and depreciation (replacement investment), the amount of net investment is $gKx$ and the depreciated part is expressed by $\widehat{\psi}(g)Kx$. We try to estimate $K$ from the angle of the marginal ratio.

Let $\Delta K$ denote the net investment matrix, and $\Delta X$, the incremental vector of output. The marginal capital coefficient $k^*_{ij}$ will be defined by:

$$k^*_{ij} = \frac{\Delta K_{ij}}{\Delta X_j}. \tag{7.2.6}$$

Assume that $k_{ij} = k^*_{ij}$. Further, we can consider the incremental output as $\Delta X = gX$.

Since the ratio $\gamma_i$ of the net investment to gross investment can be defined as follows,

$$\gamma_i = 1 - \frac{1}{(1+g)^{\tau_i}}, \tag{7.2.7}$$

we can consider the net investment matrix as $\widehat{\gamma}S$, where $S$ is the gross investment matrix of fixed capital, and $\widehat{\gamma}$ is the diagonal matrix with $\gamma_i$ in the diagonal.[5]

From this, we can set the marginal capital coefficient $k^*_{ij}$ as follows.

$$k^*_{ij} = \frac{\gamma_i S_{ij}}{gX_j}. \tag{7.2.8}$$

In this computation, $k^*_{ij}$ is dependent on the uniform growth rate $g$.

Equation (7.2.5) is rewritten as (7.2.9) with $K^*(g) = (k^*_{ij})$.

$$x = M(g)x, \tag{7.2.9}$$

$$M(g) = (\widehat{\psi}(g) + gI)K^*(g) + (1+g)A. \tag{7.2.10}$$

If $\lambda_{M(g)} = 1$, $g$ at that point gives the maximum growth rate $g^*$.

The computational procedure of $g^*$ is described below.

(1) Take a sufficiently small initial value $g_0 > 0$, and $M(g)$ in (7.2.10) is positive.

(2) For $g > 0$, we can confirm the following. From $\frac{d}{dg}M(g) > O$, $M(g)$ is an increasing function of $g$. Besides, from the Perron-Frobenius theorem, $\lambda_{M(g)}$ is an increasing function of elements of $M(g)$. Hence,

$$g_t < g_{t+1} \iff M(g_t) < M(g_{t+1}) \iff \lambda_{M(g_t)} < \lambda_{M(g_{t+1})}.$$

Let $g_{max} = \frac{1}{\lambda_A} - 1$. From $\mid g_t \mid < g_{max}$, $g_t$ is bounded.

[5]It should be noted here that $\gamma_i(g) = \gamma_i(r)$. In an economy that disregards non-productive consumption, the uniform profit rate $r$ and the uniform growth rate $g$ are equal. For details, see *e.g.* Fujimori (1982).

Now, if $\lambda_{M(g)} < 1$, the value of $g$ should be increased; if $\lambda_{M(g)} > 1$, the value of $g$ will be decreased.

A sequence $\{g_0, g_1, g_2, \ldots\}$ is generated by

$$g_{t+1} = \delta(g_t) = g_t + \beta(1 - \lambda_{M(g_t)}), \tag{7.2.11}$$

where $\beta > 0$ represents an arbitrary constant.

If $g_t$ is taken near 0, then $g_{t+1}$ will appear above the 45° line. Since the slope of $\delta(g)$ is given by

$$\frac{dg_{t+1}}{dg_t} = 1 - \beta \frac{d}{dg_t} \lambda_{M(g_t)} < 1, \tag{7.2.12}$$

$\delta(g_t)$ will cross the 45° line from the top to the bottom.

The relationship between $g_{t+1}$ and $g_t$ can be expressed as in Fig. 7.1. In view of the above, $g_{t+1} = \delta(g_t)$ has a fixed point $g_{t+1} = g_t = g^*$. $g_{t+1} = g_t = g^*$ is equivalent to $1 - \lambda_{M(g_t)} = 0$. Moreover, this fixed point is stable from (7.2.12). This fixed point can be found by the regula falsi method of numerical computation. (See *e.g.* Traub (1964) and Ortega and Rheinboldt (2000)).

(3) For $g^*$, the marginal fixed capital coefficient matrix can be constructed as follows:

$$K^* = \left(k^*_{ij}(g^*)\right) \tag{7.2.13}$$

The procedure described in the above may look like the one to compute the maximum growth rate in the golden rule of the von Neumann model. Some may argue

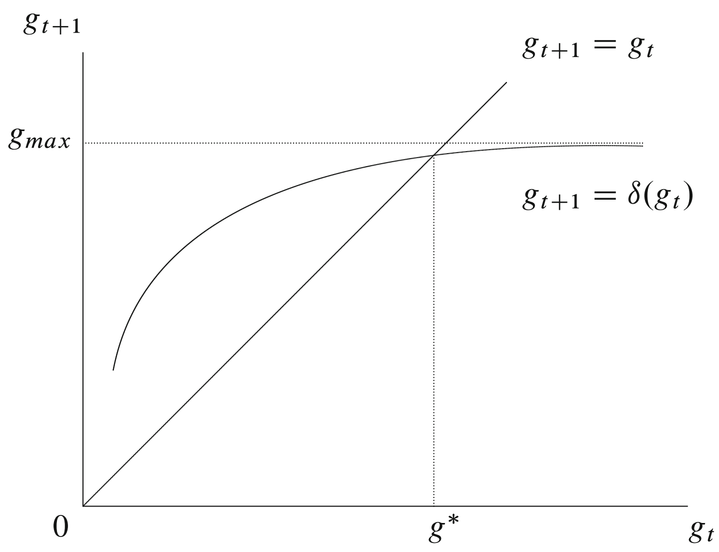


**Fig. 7.1** The convergence of the iteration

that the maximum growth rate of the von Neumann growth model is conceptually different from the maximum profit rate of Sraffa's standard system. Nonetheless, they are formally equivalent. In the same manner to estimate the maximum growth rate we evaluate the wage-profit curve in the next section, the concept of which comes from Sraffa (1960).

### 7.2.3 Notes on Data

In this chapter, the gross investment matrix of each year of China is aggregated to the matrix of the same size with similar sectors: the 33-sector input-output tables for 1987, 1990, 1992, 1995 and the 40-sector input-output tables for 1997, 2000 are aggregated to the 24-sector input-output tables. The input-output table data employed

**Table 7.1** Codes and durabilities of 24 sectors

| Code | Sectors | Durabilities (years) |
|---|---|---|
| 1 | Agriculture | 16 |
| 2 | Mining | |
| 3 | Foods and tabacco | |
| 4 | Textiles | |
| 5 | Pulp and papers | |
| 6 | Electricity, steam and hot water | |
| 7 | Petroleum and coal | |
| 8 | Coal gas and coal product | |
| 9 | Chemicals | |
| 10 | Nonmetallic mineral products | |
| 11 | Metals smelting and processing | |
| 12 | Metal products | 12 |
| 13 | General machinery | 17 |
| 14 | Transportation machinery | 9 |
| 15 | Electric machinery | 17 |
| 16 | Precise machinery | 15 |
| 17 | Other manufactured products | 12 |
| 18 | Construction | 40 |
| 19 | Transportation | 13 |
| 20 | Commercial | 10 |
| 21 | Services | |
| 22 | Finance, insurance and real estate | |
| 23 | Education, health and scientific research | |
| 24 | Public administration | |

*Note* Blanks in the durability column indicate that goods concerned are non-durable

**Table 7.2** Basic macro parameters (1987–2000)

| | 1987 | 1990 | 1992 | 1995 | 1997 | 2000 |
|---|---|---|---|---|---|---|
| $g^*$ (%) | 40.3 | 35.5 | 31.9 | 30.3 | 30.5 | 26.6 |
| $\kappa$ | 0.873 | 1.003 | 1.420 | 1.587 | 1.397 | 2.195 |

were published by the National Bureau of Statistics of China (NBSC), and the investment matrix data were estimated by Lü (2007). The durability data of fixed capital were published by Ministry of Finance of China (1992) and State Council of China (2007). The details of the input-output tables and durability of fixed capital are shown in Table 7.1.

The data of the total working population are from the NBSC publication, *China Statistical Yearbook 2003*, and the data on the annual working hours per head are from the International Labor Office (ILO) publication, *Yearbook of Labour Statistics 2003*.

From the input-output tables, the macro marginal capital-output ratio $\kappa$ of each year can be evaluated by

$$\kappa = \frac{\sum_i \sum_j \gamma_i S_{ij}}{g \sum_j X_j}.$$

The maximum potential growth rates $g^*$ of China's economy and the macro marginal capital-output ratios $\kappa$ are shown in Table 7.2.

## 7.3 Wage-Profit Curves *à la* von Neumann-Leontief

### *7.3.1 Basic Concept*

In the normal production process, many factors are employed, such as raw material and fixed capital with various durabilities and ages. Brand-new fixed capital and aged fixed capital are considered as distinctly different items. Moreover, plural types of commodities are jointly produced by processes. In a von Neumann system, the equilibrium price problem of such an economy can be described as follows[6]:

$$\max \left\{ pF \;\middle|\; \frac{1}{1+r} pB \leq pA + L,\; p \geq \mathbf{0} \right\}, \tag{7.3.1}$$

[6]See von Neumann (1945/46[1937]) for details of original von Neumann Model.

where $A$, $B$, $F$, $L$, $r$, and $p$ represent tentatively a rectangular input matrix, a rectangular output matrix, a bundle of wage goods, a labour input vector, the uniform profit rate, and the production price vector, respectively. This is a linear programming problem in which the wage rate $w = pF$ is maximised. (See Fujimoto (1975)).

Let $x$ stand for the activity level; the dual problem of (7.3.1) is expressed as follows; assuming a uniform growth rate $g = r$:

$$\min \left\{ Lx \;\middle|\; \frac{1}{1+r} Bx \geq Ax + F,\ x \geq \mathbf{0} \right\}. \tag{7.3.2}$$

This dual problem minimises the labour input in the economy.

If the above general joint-production system is looked at as a joint production system, in which aged fixed capital alone is jointly produced, then by applying the same procedure of reduction applied by Sraffa-Okishio-Nakatani, we can obtain systems of inequalities similar to (7.2.1)–(7.2.4), which will be called the *von Neumann-Leontief* system.[7]

In short, in a von Neumann-Leontief-type economy, the standard maximum problem (7.3.1) is expressed as follows.

$$\max \left\{ pF \;\middle|\; \frac{1}{1+r} p \leq pA + L + p\left( \frac{r}{1+r} I + \frac{1}{1+r} \widehat{\psi}(r) \right) K,\ p \geq \mathbf{0}_m \right\}, \tag{7.3.3}$$

where notations are the ones introduced in Sect. 7.2.

The relationship between profits and real wages can be expressed as that between the profit rate and the number of units of the bundle of wage goods. Hence, the wage-profit curves should be expressed as a curve composed of points $(\frac{1}{pF},\ r)$.

Similarly, the dual problem of (7.3.3) is expressed as follows, assuming $g = r$, *i.e.*, no capitalist-consumption:

$$\min \left\{ Lq \;\middle|\; \frac{1}{1+r} q \geq Aq + F + \left( \frac{r}{1+r} I + \frac{1}{1+r} \widehat{\psi}(r) \right) Kq,\ q \geq \mathbf{0}^m \right\}, \tag{7.3.4}$$

where $q$ represents the output vector.

[7]Refer to Fujimori (1992b) for the detailed procedure of the abridgement. Fujimori (1992b)'s method is related to Sraffa in two ways: in the way to deal with fixed capital, and in other way in the similarity between estimating the marginal fixed capital coefficient and drawing the wage-profit curves. Fujimori (1992b) was strongly conscious of Sraffa's system. Incidentally, Pasinetti (1977, Chap. VI) and especially Schefold (1980) mentioned the conceptual difference of von Neumann model, linear programming and Sraffa system, and indeed the concept of the wage-profit curve is closer to Sraffa than to von Neumann.

Linear programming problems of this kind should be considered from both short- and long-run perspectives. While the replacement and the net investment of fixed capital are generally carried out in the long run, these might be ignored in the short-run. Therefore, the short-run version of (7.3.3) can be expressed as follows.

$$\max\left\{pF \;\middle|\; \frac{1}{1+r}p \leq pA + L,\ p \geq \mathbf{0}\right\}. \tag{7.3.5}$$

The dual problem in the short run will be expressed as follows:

$$\min\left\{Lq \;\middle|\; \frac{1}{1+r}q \geq Aq + F,\ q \geq \mathbf{0}\right\}. \tag{7.3.6}$$

This corresponds to the uniform growth equilibrium with $g = r$.

### *7.3.2 Computation Procedure with Respect to Input-Output Data*

(1) Calculate the bundle of wage goods $F$ and the labour input vector $L$.
The product of the annual total working population $N_0$ and annual working hours $h$ per head gives the total working hours in a year; that is,

$$H = N_0 h.$$

The bundle of wage goods per head is the consumption divided by the total working population. In other words, this is equal to $\dfrac{C_i}{N_0}$. As for wage goods per unit of labour, one may write

$$f_i = \frac{C_i}{H}.$$

The bundle of wage goods is then given by $F = (f_i)$.
Further, the total added value $V_0$ is expressed by

$$V_0 = \sum_{j=1}^{n} W_j + \sum_{j=1}^{n} V_j + \sum_{j=1}^{n} T_j,$$

where $W_j$, $V_j$ and $T_j$ stand for wages, profits and taxes of an input-output table respectively. The working hours per unit of value is evaluated by $\dfrac{H}{V_0}$, and the working hours in sector $j$ is given by $(W_j + V_j + T_j)\dfrac{H}{V_0}$. Hence, the labour input necessary for producing one unit of goods becomes

$$l_j = \frac{H(W_j + V_j + T_j)}{V_0 X_j}.$$

The labour input vector is represented by $L = \left(l_j\right)$.

(2) Find the optimum solution for the long-run linear programming problem, and draw the wage-profit curve.
For each $r$ in the range $0 \leq r \leq g^*$, solve the long-run standard maximum problem (7.3.3), and find the optimum solution $p^*$.
The long-run wage-profit curve can be represented by a curve composed of points $(\frac{1}{p^* F}, r)$.

(3) Find the optimum solution for the short-run linear programming problem, and draw the wage-profit curve.
Repeat the same procedure of the long run as in the above with respect to (7.3.5).
Similarly, the short-run wage-profit curve is given by a curve composed of points $(\frac{1}{p^{**} F}, r)$.

(4) Estimate the coordinates of the position of actual China's economy.

(i) Since NBSC publishes $GDP$, total wage amount $\Theta^*$, and the total amount of capital formation $S^*$, the amount of total profit can be evaluated by

$$\Pi^* = GDP - \Theta^*,$$

and the accumulation rate is computed by

$$\alpha^* = \frac{S^*}{\Pi^*}.$$

(ii) For the economy containing fixed capital, the profit rate is estimated as

$$r^* = \frac{g^*}{\alpha^*}$$

from $\alpha^*$ evaluated in (i), and the growth rate $g^*$ published by NBSC.

(iii) From $r^*$, the optimal solutions for the above-mentioned LP problem (7.3.3) and (7.3.5) can be calculated, and points $(\frac{1}{p^* F}, r^*)$ and $(\frac{1}{p^{**} F}, r^*)$ on the wage-profit curve can be identified.

(iv) From consumption $C_i^*$ (with government consumption included), the wage goods per unit of labour is computed as

**Table 7.3** The long-run wage-profit relationships (1987–2000)

| $r$ | $1/pF$ | | | | | |
|---|---|---|---|---|---|---|
| | 1987 | 1990 | 1992 | 1995 | 1997 | 2000 |
| 0.00 | 1.790 | 1.849 | 1.935 | 1.904 | 1.958 | 1.889 |
| 0.02 | 1.701 | 1.752 | 1.815 | 1.781 | 1.836 | 1.754 |
| 0.04 | 1.613 | 1.656 | 1.695 | 1.658 | 1.715 | 1.619 |
| 0.06 | 1.526 | 1.561 | 1.577 | 1.535 | 1.593 | 1.482 |
| 0.08 | 1.441 | 1.466 | 1.459 | 1.413 | 1.472 | 1.345 |
| 0.10 | 1.356 | 1.372 | 1.341 | 1.291 | 1.351 | 1.207 |
| 0.12 | 1.272 | 1.278 | 1.224 | 1.169 | 1.230 | 1.068 |
| 0.14 | 1.188 | 1.184 | 1.107 | 1.047 | 1.108 | 0.929 |
| 0.16 | 1.105 | 1.089 | 0.989 | 0.925 | 0.985 | 0.788 |
| 0.18 | 1.022 | 0.993 | 0.872 | 0.802 | 0.860 | 0.645 |
| 0.20 | 0.939 | 0.896 | 0.753 | 0.678 | 0.734 | 0.501 |
| 0.22 | 0.855 | 0.797 | 0.633 | 0.552 | 0.604 | 0.353 |
| 0.24 | 0.771 | 0.695 | 0.511 | 0.424 | 0.472 | 0.201 |
| 0.26 | 0.686 | 0.590 | 0.387 | 0.293 | 0.334 | 0.045 |
| 0.28 | 0.600 | 0.480 | 0.260 | 0.158 | 0.191 | |
| 0.30 | 0.511 | 0.364 | 0.130 | 0.018 | 0.040 | |
| 0.32 | 0.420 | 0.240 | | | | |
| 0.34 | 0.326 | 0.107 | | | | |
| 0.36 | 0.228 | | | | | |
| 0.38 | 0.124 | | | | | |
| 0.40 | 0.014 | | | | | |

$$f_i^* = \frac{C_i^*}{H},$$

where $F^* = (f_i^*)$. The coordinates of the actual economy are estimated as $(\frac{1}{p^* F^*}, r^*)$.

### 7.3.3 Computation Results

The coordinates data of long- and short-run wage-profit curves *à la* von Neumann-Leontief are shown in Tables 7.3 and 7.4, respectively. The estimated profit rate and real wage rate of China's economy are shown in Table 7.5. Remark that, for computational reasons, some of the maximum profit rate cannot be numerically computed.

**Table 7.4** The short-run wage-profit relationships (1987–2000)

| $r$ | $1/pF$ | | | | | |
|---|---|---|---|---|---|---|
| | 1987 | 1990 | 1992 | 1995 | 1997 | 2000 |
| 0.00 | 1.868 | 1.929 | 2.103 | 2.065 | 2.096 | 2.062 |
| 0.05 | 1.685 | 1.729 | 1.867 | 1.825 | 1.854 | 1.813 |
| 0.10 | 1.518 | 1.546 | 1.649 | 1.604 | 1.631 | 1.582 |
| 0.15 | 1.365 | 1.376 | 1.447 | 1.399 | 1.425 | 1.367 |
| 0.20 | 1.223 | 1.219 | 1.258 | 1.208 | 1.232 | 1.166 |
| 0.25 | 1.093 | 1.071 | 1.080 | 1.028 | 1.051 | 0.976 |
| 0.30 | 0.971 | 0.932 | 0.911 | 0.858 | 0.879 | 0.796 |
| 0.35 | 0.857 | 0.799 | 0.751 | 0.696 | 0.715 | 0.622 |
| 0.40 | 0.750 | 0.672 | 0.596 | 0.540 | 0.557 | 0.454 |
| 0.45 | 0.648 | 0.548 | 0.445 | 0.389 | 0.404 | 0.290 |
| 0.50 | 0.552 | 0.426 | 0.298 | 0.241 | 0.252 | 0.128 |
| 0.55 | 0.460 | 0.304 | 0.152 | 0.095 | 0.102 | |
| 0.60 | 0.371 | 0.179 | 0.006 | | | |
| 0.65 | 0.285 | 0.050 | | | | |
| 0.70 | 0.201 | | | | | |
| 0.75 | 0.117 | | | | | |
| 0.80 | 0.034 | | | | | |
| $g_{max}$ | 0.820 | 0.668 | 0.602 | 0.582 | 0.583 | 0.539 |

**Table 7.5** The profit rate and real wage rate in China's economy (1987–2000)

| | | 1987 | 1990 | 1992 | 1995 | 1997 | 2000 |
|---|---|---|---|---|---|---|---|
| Profit rate | $r^*$ | 0.216 | 0.119 | 0.291 | 0.215 | 0.203 | 0.194 |
| Real wage rate | $\frac{1}{p^{**}F}$ | 1.182 | 1.479 | 0.941 | 1.152 | 1.223 | 1.191 |
| | $\frac{1}{p^*F}$ | 0.874 | 1.281 | 0.190 | 0.582 | 0.719 | 0.546 |
| | $\frac{1}{p^*F^*}$ | 0.748 | 1.116 | 0.160 | 0.521 | 0.634 | 0.478 |

The wage-profit curves from 1987 to 2000 (W-P curve) based on the above-mentioned tables are drawn as in Figs. 7.2, 7.3, 7.4, 7.5, 7.6 and 7.7. The short-run curves are shown for comparison.

The coordinates of the estimated position of actual China's economy are indicated with "⊕" in the figures of the wage-profit curves.

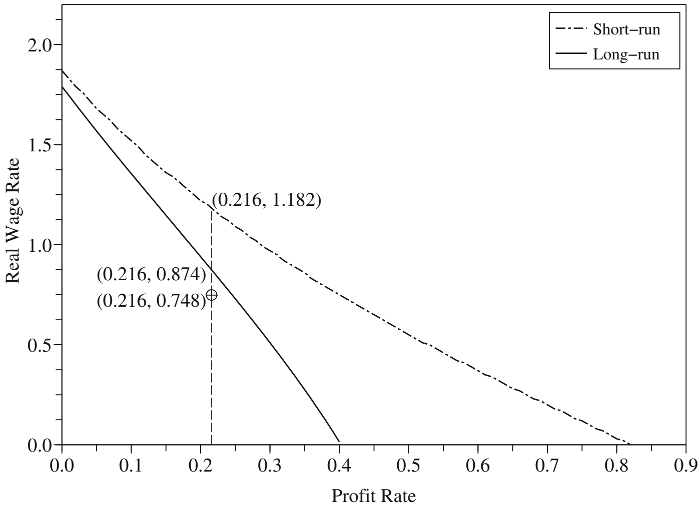


**Fig. 7.2** The wage-profit curve *à la* von Neumann-Leontief (1987)

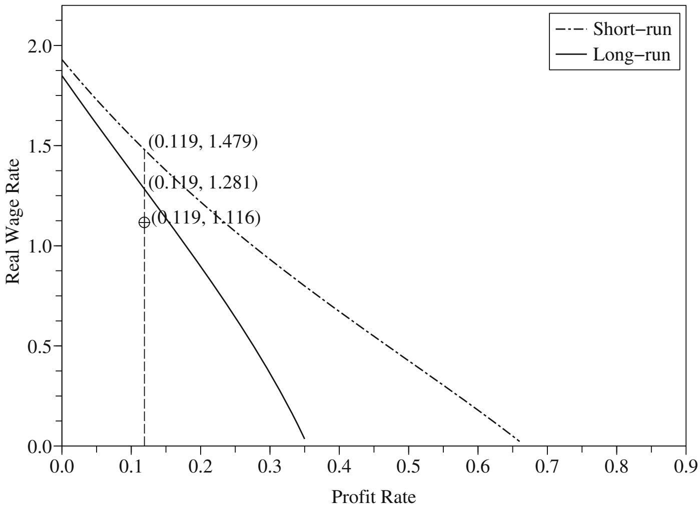


**Fig. 7.3** W-P curve (1990)

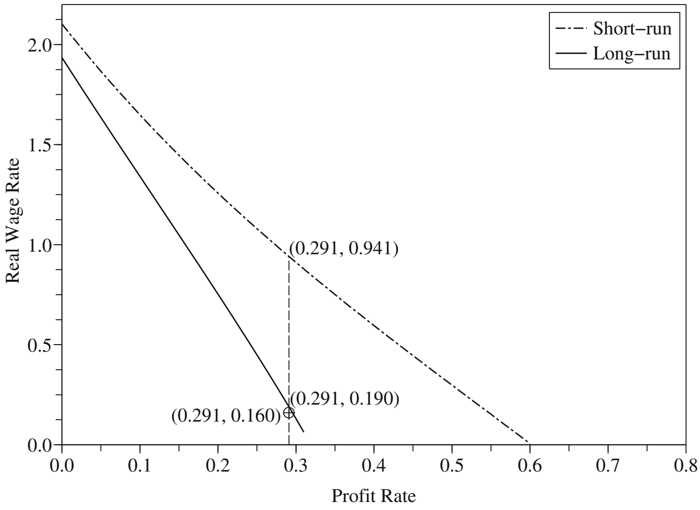


**Fig. 7.4** W-P curve of 1992

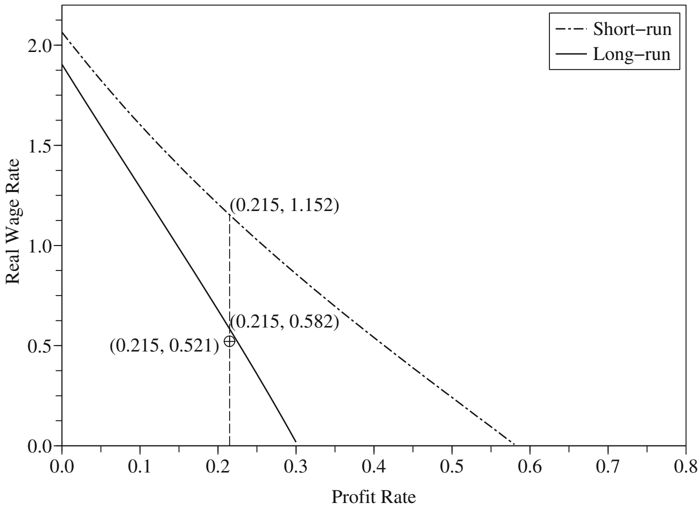


**Fig. 7.5** W-P curve of 1995

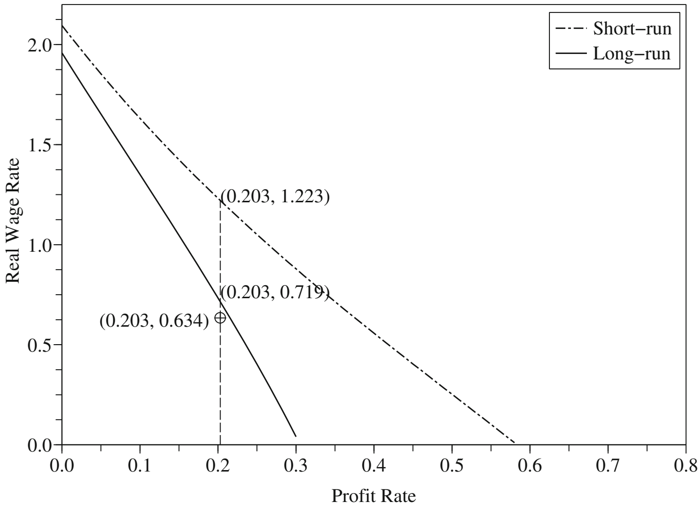


**Fig. 7.6** W-P curve of 1997

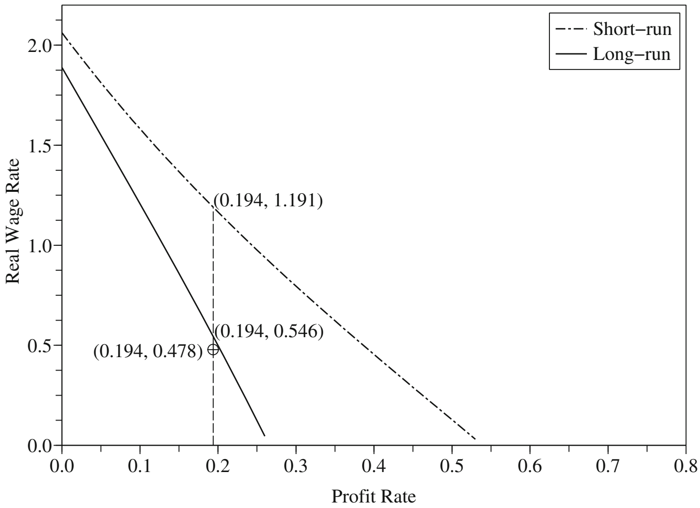


**Fig. 7.7** W-P curve of 2000

## 7.4 Concluding Remarks

In this chapter, we estimated China's marginal fixed capital coefficients by means of the Sraffa-Fujimori method, and computed the wage-profit curves in a von Neumann-Leontief system.

The following points concerning China's economy are clearly seen in the period of 1980s, 1990s and 2000s from our computation of theoretical parameters.

(1) Both the short-run maximum profit rate and the long-run maximum potential growth rate tends to decrease over the period.
(2) The maximum real wage rate increased over the period, both short- and long-run, although slight fluctuations are observed.
(3) As the rate of profit increases, the long-run wage-profit curves tend to sink faster than the case of short run in each year.
(4) The marginal fixed capital-output ratio has increased.

In this chapter, various theoretical restrictions, such as physical durability of fixed capital with constant efficiency *etc.*, exist. However, estimating theoretical values by using actual data, and further making simulations revealed significance of fixed capital data for better understanding of the performance of the actual economy.

In view of Government's policy to raise wages during period concerned, it is expected that the real wages have been increasing monotonically. However, Table 7.5 shows only the rising tendency. This should be investigated further in future.

# Chapter 8
# Marx's Labour Values in China's Economy

## 8.1 Introduction

As well-known, Marx started from the concept of value based on the amount of abstract human labour, and established the production price system via the so-called transformation procedure, values into prices and surplus value into profit.

Some argues that the production price system is self-explanatory, that is, the production price system does not need the value basis, as insisted by the transformation. However, the relative prices of production may be determined without taking values into account; the absolute prices cannot be determined.

Marx's theory of values and production prices shows that labour values and production prices of commodities are important magnitudes reflecting the structure of capitalist systems of commodity production.

With regard to computation of ratios of production prices to values, the rate of surplus value and the organic composition of capital, many studies and analyses have been done, such as, Okishio (1959), Okishio and Nakatani (1975, 1993), Nakatani (1976), Wolff (1979), Parys (1982), Ochoa (1989), and further in the recent years, Tsoulfidis and Rieu (2006), Tsoufidis and Mariolis (2007), Tsoulfidis (2008), Mariolis and Tsoulfidis (2009, 2010), Mariolis (2011). All these studies deal with the economy without fixed capital.

In this chapter, we compute labour values, production prices and investigate some implications with respect to China's economy. The basic framework and various parameters come from preceding chapters.

In the first place, we compute values and production prices of commodities in a fixed-capital economy, in which aged fixed capital is treated as joint-products.

Since the production prices computed in what follows are measured in terms of amount of labour, we can argue unequal exchange of labour between sectors by comparing values and prices. In normalising the production prices, we take into account one equality proposition of the transformation problem. We compute the ratios of prices to values, and examine the relations between price-value ratios and the organic composition of capital.



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Secondly, we compute the organic composition of capital of each sector. Since fixed capital is included in our model, we additionally compute the fixed capital-labour ratio of each sector.

Lastly, we compute two types of Spearman's rank correlation coefficients of the rank of the ratios and organic composition of capital or fixed capital-labour ratios of the sectors. That is, we carry out Spearman's rank correlation coefficient analysis to the combination of the ranks of value-price ratios and the ranks of organic compositions of capital and to that of the ranks of value-price ratios and the ranks of fixed capital-labour ratios.

## 8.2 Value System and Production Price System

We summarise here the system of symbols and the framework of analysis.

### *8.2.1 Notations and Preliminary Notes*

Let $K$ stand for the fixed capital coefficient matrix of a SON economy. Let $A$, $L$ and $F$ be the input coefficient matrix, the labour input vector, and the consumption goods bundle, respectively. Let $w$, $p$ and $x$ stand for the value vector, the production price vector and the output vector, respectively.

The rate of depreciation of fixed capital is a function of the profit rate $r$, and it is expressed by $\psi(r) = \dfrac{1}{\sum_{j=0}^{\tau-1}(1+r)^j}$.

As for the existence of positive values and production prices, we assume that the existence is guaranteed together with the existence of positive profit rate.

### *8.2.2 Values*

The value system *à la* SON can be expressed as follows:

$$w = w(\widehat{\psi}(0)K + A) + L, \tag{8.2.1}$$

$$\widehat{\psi}(0) = \mathrm{diag}\left(\frac{1}{\tau_1}, \frac{1}{\tau_2}, \ldots, \frac{1}{\tau_n}\right). \tag{8.2.2}$$

For a non-negative matrix $(\widehat{\psi}(0)K + A)$, we can obtain the absolute value of $w^*$ by the following equation from (8.2.1);[1]

[1]For $(\widehat{\psi}(0)K + A) \geq O$, the nonsingularity and the reversibility of $(I - \widehat{\psi}(0)K - A)$ are known, and $(I - \widehat{\psi}(0)K - A)^{-1} \geq O$ is obvious from the Perron-Frobenius theorem.

$$w^* = L\big(I - \widehat{\psi}(0)K - A\big)^{-1}. \tag{8.2.3}$$

### 8.2.3 Production Prices

The production price system *à la* SON is expressed as follows:

$$p = pM(r), \tag{8.2.4}$$
$$M(r) = (\widehat{\psi}(r) + rI)K + (1 + r)(A + FL). \tag{8.2.5}$$

A non-negative matrix $M(r)$ should have the Perron-Frobenius eigenvalue 1 and the Perron-Frobenius eigenvector $p^1$ corresponding to 1, and the corresponding uniform rate of profit $r^*$ is determined accordingly.

In normalising the production prices of commodities, we employ Marx's equality of total values and total prices. That is, we assume that the production prices are determined at their absolute levels by finding the multiplier $\alpha$ that satisfies the following.

$$wx = \alpha p^1 x. \tag{8.2.6}$$

Then, the absolute production price can be determined as follows:

$$p^* = \alpha p^1. \tag{8.2.7}$$

Therefore, we can estimate production price-value ratio $\varepsilon = (\varepsilon_i)$ by

$$\varepsilon_i = \frac{p_i^*}{w_i^*}. \tag{8.2.8}$$

### 8.2.4 Rate of Surplus Value

The rate of surplus value $\mu$ can be obtained by the following equation including fixed capital.

$$\mu = \frac{M_i}{V_i} = \frac{w(I - \widehat{\psi}(0)K - A - FL)x}{wFLx} = \frac{1}{wF} - 1. \tag{8.2.9}$$

The results of computation are shown in Table 8.1.

**Table 8.1** Time series of production price-value ratio

| Code | Sector name | 1987 | 1990 | 1992 | 1995 | 1997 | 2000 | Average |
|---|---|---|---|---|---|---|---|---|
| 1 | Agriculture | 0.7657 | 0.7300 | 0.7468 | 0.7619 | 0.7465 | 0.7724 | 0.7543 |
| 2 | Mining | 1.0419 | 1.1080 | 1.0823 | 1.0214 | 0.9994 | 0.9533 | 1.0316 |
| 3 | Foods and tobacco | 0.9278 | 0.8705 | 0.8958 | 0.8434 | 0.8690 | 0.8643 | 0.8698 |
| 4 | Textiles | 1.0576 | 1.0438 | 1.0687 | 1.0602 | 0.9663 | 0.9780 | 1.0507 |
| 5 | Pulp and papers | 1.0336 | 1.0462 | 1.0405 | 1.0382 | 1.0022 | 1.0249 | 1.0359 |
| 6 | Electricity, steam and hot water | 1.1799 | 1.2505 | 1.2099 | 1.1548 | 1.0846 | 1.0515 | 1.1674 |
| 7 | Petroleum and coal | 0.9921 | 1.1261 | 1.1393 | 1.0796 | 1.0698 | 1.0011 | 1.0747 |
| 8 | Coals gas and coal product | 1.2539 | 1.2251 | 1.2175 | 1.2377 | 1.3859 | 1.1920 | 1.2314 |
| 9 | Chemicals | 1.1090 | 1.1158 | 1.0842 | 1.0859 | 1.0505 | 1.0593 | 1.0851 |
| 10 | Nonmetallic mineral products | 1.0360 | 1.0767 | 1.0493 | 1.0530 | 1.0516 | 1.0548 | 1.0523 |
| 11 | Metals smelting and processing | 1.1407 | 1.2119 | 1.0966 | 1.1008 | 1.1832 | 1.1334 | 1.1371 |
| 12 | Metal products | 1.0936 | 1.1405 | 1.0957 | 1.0907 | 1.1378 | 1.1061 | 1.1009 |
| 13 | General machinery | 1.0944 | 1.1531 | 1.0727 | 1.0727 | 1.0536 | 1.0758 | 1.0743 |
| 14 | Transportation machinery | 1.1610 | 1.2014 | 1.0947 | 1.1122 | 1.1073 | 1.0989 | 1.1097 |
| 15 | Electric machinery | 1.1483 | 1.1911 | 1.0981 | 1.0765 | 1.1244 | 1.0995 | 1.1120 |
| 16 | Precise machinery | 1.0849 | 1.1385 | 1.0580 | 1.0696 | 1.0630 | 1.0716 | 1.0706 |
| 17 | Other manufactured products | 1.2074 | 1.1694 | 1.1159 | 1.2023 | 0.9986 | 1.0600 | 1.1427 |
| 18 | Construction | 1.0773 | 1.0823 | 1.0344 | 1.0278 | 1.0382 | 1.0193 | 1.0363 |
| 19 | Transportation | 1.1029 | 1.0170 | 1.1014 | 1.1510 | 1.2702 | 1.2135 | 1.1270 |
| 20 | Commercial | 1.0563 | 1.0921 | 0.9498 | 0.9047 | 0.9444 | 0.9251 | 0.9471 |
| 21 | Services | 1.0165 | 1.0279 | 0.9493 | 0.9819 | 1.0223 | 0.9802 | 0.9992 |
| 22 | Finance, insurance and real estate | 0.7495 | 0.7555 | 0.9050 | 0.9583 | 0.9726 | 0.9379 | 0.9215 |
| 23 | Education, health and science | 1.0282 | 1.0063 | 0.9213 | 0.9445 | 0.9504 | 0.9303 | 0.9475 |
| 24 | Public administration | 1.2073 | 1.2233 | 1.1377 | 1.1959 | 1.2121 | 1.2030 | 1.2051 |
| Total | | 25.5658 | 26.0029 | 25.1651 | 25.2250 | 25.3040 | 24.8063 | – |
| Uniform rate of profit | | 0.1852 | 0.1785 | 0.1581 | 0.1477 | 0.1574 | 0.1297 | – |
| Rate of surplus value | | 0.7901 | 0.8495 | 0.9349 | 0.9043 | 0.9583 | 0.8890 | – |

## 8.3 Organic Composition of Capital, Fixed Capital-Labour Ratio and Rate of Surplus Value

In this section, we compute two types of magnitudes that will be regarded as Marx's organic composition of capital or its extension.

### 8.3.1 Organic Composition of Capital

Let $C$ and $V$ stand for constant capital and variable capital, respectively. We then obtain the ratio $\frac{C_i}{V_i}$ of sector $i$, which is called the *organic composition of capital* by Marx. Here,

$$C = w\big(\widehat{\psi}(0)K + A\big), \tag{8.3.1}$$

$$V = wFL, \tag{8.3.2}$$

represent the constant and the variable capital, respectively.

The organic composition of capital $\xi = (\xi_i)$ is obtained as follows:

$$\xi_i = \frac{C_i}{V_i}. \tag{8.3.3}$$

Remark that we take up annual flows of inputs, *i.e.* depreciation of fixed capital and non-durable means of production.

### 8.3.2 Fixed Capital-Labour Ratios

According to the calculation of fixed capital based on value $\mathcal{K}(= wK)$, the fixed capital-labour ratio $\delta$ can be expressed as follows:

$$\delta_i = \frac{\mathcal{K}_i}{L_i}. \tag{8.3.4}$$

Remark that we do not include the inputs of non durable means of production.

The results are shown in the following Table 8.4.

## 8.4 Spearman's Rank Correlation Coefficients

In Marx's simple 2-sector model of reproduction without fixed capital, one can infer that the production price of commodities in terms of labour, which is produced with higher organic composition of capital, exceeds the value of the commodities. In a multi-sector model, however, it is not easy to derive a similar theoretical result.

In stead of the theoretical analysis, we try to examine if there is a correlation between the price-value rank and the rank of organic composition of capital of sectors. In addition, we consider the rank of fixed-capital-labour ratios of sectors.

The results are summarised in the following Table 8.5.

## 8.5 Concluding Remarks

So far, we computed the values of commodities in the framework of 24 major sectors of China's economy. Our observations will be summarised as follows:

(1) Tendency of the (uniform) rate of profit to fall is remarkable from 1980s to 2000. As for this, the increase in the investment in fixed capital originates every year.
(2) It is the tendency for the rate of surplus value to go up from 1980s to 2000.
(3) Tables 8.1 and 8.2 show that the production prices in sectors 5 (pulp and papers), 6 (electricity, steam and hot water), 8 (coals gas and coal product), 9 (chemicals), 10 (nonmetallic mineral products), 11 (metals smelting and processing), 12 (metal products), 13 (general machinery), 14 (transportation machinery), 15 (electric machinery), 16 (precise machinery), 18 (construction), 19 (transportation), 24 (public administration) are larger than values, whereas the production prices in sectors 1 (agriculture), 3 (foods and tobacco), 22 (finance, insurance and real estate) are smaller than values, even if it takes the averages from 1980s to 2000

**Table 8.2** The comparison of production price and value

| | |
|---|---|
| $p^* > w^*$ (1987–2000) | 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 24 |
| $p^* > w^*$ (Average) | 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 24 |
| $p^* < w^*$ (Average) | 1, 3, 20, 21, 22, 23 |
| $p^* < w^*$ (1987–2000) | 1, 3, 22 |
| Rank of sectors in average | 8, 24, 6, 17, 11, 19, 15, 14, 12, 9, 7, 13, 16, 10, 4, 18, 5, 2, 21, 23, 20, 22, 3, 1 |

**Table 8.3** Time series of organic composition of capital

| | 1987 | 1990 | 1992 | 1995 | 1997 | 2000 |
|---|---|---|---|---|---|---|
| 1 | 0.7892 | 0.9025 | 1.1038 | 1.3051 | 1.3076 | 1.3753 |
| 2 | 1.6676 | 2.8933 | 3.1841 | 2.6010 | 2.3623 | 1.8500 |
| 3 | 5.7781 | 5.7744 | 6.3294 | 3.9073 | 5.6514 | 4.6914 |
| 4 | 5.6119 | 7.1540 | 8.6086 | 8.4599 | 5.1823 | 6.0557 |
| 5 | 3.8776 | 5.1029 | 6.0622 | 6.4413 | 4.8451 | 5.9826 |
| 6 | 1.6741 | 2.5746 | 4.0289 | 3.4484 | 2.7693 | 3.2709 |
| 7 | 1.8992 | 3.9718 | 6.1596 | 6.2614 | 7.6181 | 6.4722 |
| 8 | 11.7824 | 6.6350 | 10.9869 | 30.7003 | 9.6491 | 11.1885 |
| 9 | 4.2952 | 5.2747 | 6.1036 | 6.5346 | 5.9573 | 6.9035 |
| 10 | 2.7175 | 3.7632 | 4.3108 | 4.7054 | 4.8522 | 5.4835 |
| 11 | 3.9833 | 5.7959 | 5.7838 | 6.3853 | 9.1300 | 9.3554 |
| 12 | 3.7541 | 5.0137 | 6.6356 | 6.8651 | 7.0060 | 8.2497 |
| 13 | 3.5275 | 4.8376 | 5.5190 | 5.6561 | 4.1543 | 5.7535 |
| 14 | 4.6729 | 5.5952 | 6.0185 | 6.7519 | 5.8721 | 6.7124 |
| 15 | 4.6266 | 5.6203 | 6.4775 | 5.9481 | 6.8084 | 7.4435 |
| 16 | 2.6107 | 3.9660 | 4.4095 | 4.7139 | 4.5092 | 6.0172 |
| 17 | 4.3528 | 6.1254 | 7.4448 | 8.1948 | 4.4777 | 6.3366 |
| 18 | 4.1583 | 4.1174 | 4.8200 | 4.7576 | 4.8239 | 5.1735 |
| 19 | 1.5805 | 1.4684 | 2.6421 | 2.8081 | 3.3092 | 3.6006 |
| 20 | 2.0674 | 3.1745 | 2.4622 | 1.6945 | 2.1773 | 2.4234 |
| 21 | 2.3650 | 2.7547 | 2.6239 | 2.5375 | 3.6566 | 3.6738 |
| 22 | 0.1653 | 0.2065 | 1.8837 | 1.4352 | 1.5350 | 1.2418 |
| 23 | 2.3479 | 2.3693 | 2.0039 | 2.0648 | 2.4547 | 2.1836 |
| 24 | 1.4412 | 1.4809 | 3.1086 | 3.3134 | 3.5739 | 3.4291 |

and takes an annual value. It is shown that the production price-value ratio in the agriculture is the lowest, which indicates that agriculture is facing unequal exchange of labour with other sectors.

(4) From Tables 8.3 and 8.4, we can observe that, the labour-intensive type of the agricultural sector and the capital-intensive type of the industrial sectors are known. It can be said that mechanisation in agriculture has not progressed like in other industries.

**Table 8.4** Time series of fixed capital-labour ratio

| | 1987 | 1990 | 1992 | 1995 | 1997 | 2000 |
|---|---|---|---|---|---|---|
| 1 | 0.1959 | 0.1834 | 0.2959 | 0.2471 | 0.1238 | 0.3536 |
| 2 | 1.4518 | 1.6793 | 3.0327 | 2.2796 | 1.7471 | 2.2011 |
| 3 | 0.3060 | 0.3024 | 0.6887 | 0.4608 | 0.4524 | 0.8676 |
| 4 | 0.5160 | 0.5637 | 1.2948 | 0.9517 | 0.5643 | 0.8033 |
| 5 | 0.3120 | 0.3632 | 0.7610 | 0.7469 | 0.6217 | 1.3189 |
| 6 | 3.3463 | 4.0505 | 6.4106 | 5.5310 | 3.1232 | 3.8296 |
| 7 | 0.8429 | 1.4882 | 3.1976 | 2.3726 | 0.8789 | 1.3631 |
| 8 | 3.7692 | 2.4883 | 5.8766 | 15.1397 | 14.3299 | 10.2654 |
| 9 | 1.4114 | 1.5054 | 1.6388 | 1.5655 | 0.9232 | 1.6652 |
| 10 | 0.5535 | 0.5162 | 1.1983 | 1.2856 | 0.9692 | 2.0607 |
| 11 | 1.0552 | 1.3011 | 1.2695 | 1.4935 | 2.0650 | 2.6338 |
| 12 | 0.3203 | 0.3529 | 0.5516 | 0.6501 | 0.5247 | 1.1209 |
| 13 | 0.6289 | 0.7477 | 0.8595 | 0.9363 | 0.7221 | 1.2990 |
| 14 | 1.0153 | 1.2245 | 1.4691 | 1.6275 | 0.9441 | 1.4832 |
| 15 | 0.8680 | 1.0883 | 1.2329 | 0.9318 | 0.7228 | 0.8615 |
| 16 | 1.2090 | 1.1014 | 1.7109 | 2.0120 | 0.8463 | 1.2460 |
| 17 | 2.6719 | 1.1279 | 2.6212 | 8.3607 | 0.8309 | 3.3747 |
| 18 | 0.0762 | 0.0861 | 0.2728 | 0.2670 | 0.2375 | 0.3904 |
| 19 | 2.4442 | 1.4732 | 3.9915 | 5.5386 | 7.3823 | 9.2465 |
| 20 | 2.5219 | 3.2255 | 1.0273 | 1.1012 | 1.1984 | 1.0337 |
| 21 | 1.3989 | 1.8190 | 1.2634 | 2.2901 | 2.0358 | 2.0157 |
| 22 | 0.6311 | 0.8685 | 0.8418 | 2.2181 | 2.1580 | 2.7193 |
| 23 | 0.7592 | 0.7356 | 0.8511 | 1.2373 | 0.8782 | 1.3250 |
| 24 | 3.8823 | 4.2782 | 4.8007 | 6.4836 | 5.9876 | 8.2840 |

**Table 8.5** Spearman's rank correlation coefficients

| | 1987 | 1990 | 1992 | 1995 | 1997 | 2000 |
|---|---|---|---|---|---|---|
| $\rho_1$ | 0.4947 | 0.4790 | 0.5901 | 0.5253 | 0.5826 | 0.5946 |
| $\rho_2$ | 0.6077 | 0.4953 | 0.6876 | 0.6537 | 0.7451 | 0.6601 |

*Note* $\rho_1$ and $\rho_2$ represent the Spearman's rank correlation coefficients of $(\varepsilon, \xi)$ and $(\varepsilon, \delta)$, respectively

(5) Since Spearman's rank correlation coefficients $\rho_1$ and $\rho_2$ are larger than 0.4, it becomes clear that production price-value ratio have (strong) correlativity with organic composition of capital, and fixed capital-labour ratio from Table 8.5. That is to say, production price-value ratio, organic composition of capital and fixed capital-labour ratio have the same tendency in China's economy from 1980s to 2000.

# Chapter 9
# Turnpike Paths for China's Economy 1995–2000

## 9.1 Introduction

This chapter explores the long-term planning model based on the Marx-Sraffa-von Neumann system. It allows joint production and supply-demand relationships between periods are represented by inequality conditions. The target of the plan is to maximise consumption in the economy in the last period.

The following discussion starts from Kantrovich (1959). Kantrovich (1959) presented the model called the perspective planning model. It is a model to maximise the unit of consumption basket of the final period of the perspective planning.

First, the long-term planning theory proposed by Kantorovich (1959) is transformed into the standard maximisation problem.

Next, with respect to the constraints, restrictions on fixed capital transfer is taken into consideration.

Besides, this chapter also takes up a labour resource constraint as an additional condition.

Needless to say, in the following investigation, the marginal fixed capital coefficients of China, as estimated in Chap. 7 will be applied.

## 9.2 Kantorovich's Long-Term Planning Model

Let $M$ be an input matrix, $x$ be an activity level, and $B$ be an output matrix. The amount of inputs $Mx$ is not supplied other than an initial value. If an initial value is made into $d(0)$, then

$$Mx(1) \leq d(0). \tag{9.2.1}$$

In the middle of the planning periods, we introduce the resource constraint condition that the amount of the required inputs of a certain period does not exceed the quantity of the previous period. Therefore, if the output $Bx(t)$ of period $t$ becomes



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the input of the next period, the input $Mx(t+1)$ of period $t+1$ must fullfil the following formula. Namely,

$$Bx(t) \geq Mx(t+1). \tag{9.2.2}$$

In the middle of the planning periods, only these constraints are effective. Maximisation by the predetermined ratio $\boldsymbol{a} = {}^t(\alpha_1, \ldots, \alpha_r)$ of an additional final product is given at the last period of the plan.

$$Bx(t) \geq \boldsymbol{a}k. \tag{9.2.3}$$

$k$ is the number of units of the product of the final period.

We can thus summarize the above conditions until the last period of the plan as follows:

$$\begin{pmatrix} M & O & O & \cdots & O & 0 \\ -B & M & O & \cdots & O & 0 \\ O & -B & M & \cdots & O & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ O & \cdots & O & -B & M & 0 \\ O & \cdots & \cdots & O & -B & \boldsymbol{a} \end{pmatrix} \begin{pmatrix} x(1) \\ x(2) \\ \vdots \\ \vdots \\ x(t) \\ k \end{pmatrix} \leq \begin{pmatrix} d(0) \\ 0 \\ \vdots \\ \vdots \\ 0 \end{pmatrix}. \tag{9.2.4}$$

The coefficient matrix of (9.2.4), the variable vector, and the constant vector of the right-hand side are denoted by $G$, $\boldsymbol{x}(t)$, and $\boldsymbol{d}$, respectively. Then, the constraints can be written as follows:

$$G\boldsymbol{x}(t) \leq \boldsymbol{d}. \tag{9.2.5}$$

Since the objective function is $k$, let $\boldsymbol{v}$ be

$$\boldsymbol{v} = \begin{pmatrix} 0 & \cdots & 0 & 1 \end{pmatrix} \tag{9.2.6}$$

concerning a variable vector, then the objective function can be expressed as $\boldsymbol{vx}(t)$.

Therefore, the following linear programming problem is acquired:

$$\max\{\boldsymbol{vx}(t) \mid G\boldsymbol{x}(t) \leq \boldsymbol{d}, \boldsymbol{x}(t) \geq 0\}. \tag{9.2.7}$$

In terms of the economic meaning of this problem, the maintenance plan of future capital equipment and the social-capital infrastructure improvement that influences consumption habits can be considered.

The dual problem of (9.2.7) is therefore expressed by

$$\min\{\boldsymbol{u}(t)\boldsymbol{d} \mid \boldsymbol{u}(t)G \geq \boldsymbol{v}, \boldsymbol{u}(t) \geq \circ\}, \tag{9.2.8}$$

where $\boldsymbol{u}(t)$ is a dual variable. This elucidates the meaning of the so-called shadow price, which allows us to judge the degree of achievement of the plan.

### 9.2.1 DOSSO's Model

Assume an economy with the input matrix $A$, the output matrix $B$ and the labour vector $L$. Let $F$ stand for a wage goods bundle. If the level of operation of period $t$ is denoted by $x(t)$, the output vector is given by $Bx(t)$. The input for the next period cannot exceed the output of this period, so that one has

$$Bx(t) \geqq Mx(t+1). \tag{9.2.9}$$

If this condition holds during the planning period, (9.2.9) will be extended to

$$\begin{pmatrix} M & O & O & \cdots & O & O \\ -B & M & O & \cdots & O & O \\ O & -B & M & \cdots & O & O \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ O & \cdots & O & -B & M & O \\ O & \cdots & \cdots & O & -B & M \end{pmatrix} \begin{pmatrix} x(1) \\ x(2) \\ \vdots \\ \vdots \\ x(t) \end{pmatrix} \leqq \begin{pmatrix} d(0) \\ 0 \\ \vdots \\ \vdots \\ 0 \end{pmatrix}, \tag{9.2.10}$$

where $d(0)$ gives an initial condition.

Let $G$, $\boldsymbol{d}$ and $\boldsymbol{x}(t)$ stand for the coefficient matrix, the constant vector and the variable vector of (9.2.10), respectively, and one can write

$$G\boldsymbol{x}(t) \leqq \boldsymbol{d}. \tag{9.2.11}$$

The objective function is such that maximises a scalar, with $\boldsymbol{v} = \begin{pmatrix} \circ & \cdots & \circ & 1 & \cdots & 1 \end{pmatrix}$,

$$\max\{\boldsymbol{v}\boldsymbol{x}(t) \mid G\boldsymbol{x}(t) \leqq \boldsymbol{d}, \boldsymbol{x}(t) \geqq 0\}. \tag{9.2.12}$$

The dual problem of (9.2.12) is described as

$$\min\{\boldsymbol{y}(t)\boldsymbol{d} \mid \boldsymbol{y}(t)G \geqq \boldsymbol{v}, \boldsymbol{y}(t) \geqq \circ\}, \tag{9.2.13}$$

where $\boldsymbol{y}(t) = \begin{pmatrix} p(1) & p(2) & \cdots & p(t) \end{pmatrix}$.

Remark that, with the same period of the plan, the dimension of coefficient matrices of the Kantorovich model will be larger than that of DOSSO.

### 9.2.2 Similarity and Difference Between DOSSO and Kantorovich Model

Here, we assume that the planning period comprises 2 cycles of reproduction, and we will show that the Kantorovich model is reduced to the DOSSO style, and these two are substantively the same.

First, the constraint for the DOSSO model of 2-period planning (9.2.10) is expressed as

$$\begin{pmatrix} M & O \\ -B & M \end{pmatrix} \begin{pmatrix} x(1) \\ x(2) \end{pmatrix} \leqq \begin{pmatrix} d(0) \\ 0 \end{pmatrix}. \tag{9.2.14}$$

Let $\boldsymbol{x}_d = \begin{pmatrix} x(1) \\ x(2) \end{pmatrix}$ stand for the variable vector, and $\boldsymbol{v}_d = \begin{pmatrix} \circ\, 1 \cdots 1 \end{pmatrix}$ denote the coefficient vector of the objective function, and the standard maximising problem is formulated as follows:

$$\max \left\{ \boldsymbol{v}_d \boldsymbol{x}_d \;\middle|\; \begin{pmatrix} M & O \\ -B & M \end{pmatrix} \boldsymbol{x}_d \leqq \begin{pmatrix} d(0) \\ 0 \end{pmatrix},\ \ \boldsymbol{x}_d \geqq 0 \right\}. \tag{9.2.15}$$

Second, the constraint of the Kantorovich model of this case (9.2.4) will be stated as follows:

$$\begin{pmatrix} M & O & 0 \\ -B & M & 0 \\ O & -B & \boldsymbol{a} \end{pmatrix} \begin{pmatrix} x(1) \\ x(2) \\ k \end{pmatrix} \leqq \begin{pmatrix} d(0) \\ 0 \\ 0 \end{pmatrix}. \tag{9.2.16}$$

Let $\boldsymbol{v}_k = \begin{pmatrix} \circ\, 1 \end{pmatrix}$ and $\boldsymbol{x}_k = \begin{pmatrix} x(1) \\ x(2) \\ k \end{pmatrix}$ stand for the coefficient vector of the objective function and the variable vector, respectively, and the standard maximising problem will be expressed by the following:

$$\max \left\{ \boldsymbol{v}_k \boldsymbol{x}_k \;\middle|\; \begin{pmatrix} M & O & 0 \\ -B & M & 0 \\ O & -B & \boldsymbol{a} \end{pmatrix} \boldsymbol{x}_k \leqq \begin{pmatrix} d(0) \\ 0 \\ 0 \end{pmatrix},\ \boldsymbol{x}_k \geqq 0 \right\}. \tag{9.2.17}$$

Transform (9.2.16), and one obtains

$$\begin{pmatrix} M & O \\ -B & M \end{pmatrix} \begin{pmatrix} x(1) \\ x(2) \end{pmatrix} \leqq \begin{pmatrix} d(0) \\ Bx(2) - \boldsymbol{a}k \end{pmatrix}. \tag{9.2.18}$$

Suppose that the number of units of the given proportion $\boldsymbol{a}$ of final products of the last period of planning is maximised, and

$$Bx(2) = \boldsymbol{a}k \Longrightarrow k = \boldsymbol{a}^{+} Bx(2)$$

follows.[1]

[1]Since $\boldsymbol{a} \geqq 0$ is a column vector, by applying Moore-Penrose quasi inverse (Chap. 6), one sees that $\boldsymbol{a}^{+}\boldsymbol{a} = 1\boldsymbol{a}^{+}$ is a row vector.

Thus, the objective function $\boldsymbol{v}_k\boldsymbol{x}_k$ of (9.2.17) can be replaced by $\boldsymbol{v}_a\boldsymbol{x}_d$; that is,

$$\boldsymbol{v}_a = \left(\circ\ \boldsymbol{a}^+\right)\begin{pmatrix} B & \\ & B \end{pmatrix} = \left(\circ\ \boldsymbol{a}^+ B\right).$$

Hence, Kantorovich's (9.2.17) can be replaced by the following:

$$\max\left\{\boldsymbol{v}_a\boldsymbol{x}_d \;\middle|\; \begin{pmatrix} M & O \\ -B & M \end{pmatrix}\boldsymbol{x}_d \leqq \begin{pmatrix} d(0) \\ 0 \end{pmatrix},\ \boldsymbol{x}_d \geqq 0\right\}. \tag{9.2.19}$$

Clearly, (9.2.19) and (9.2.15) are of the similar pattern.[2]

Since $\boldsymbol{v}_a$ can vary according to $\boldsymbol{a}$, from the above, one may regard that DOSSO's turnpike model and Kantorovich's planning model are not different from each other.

As afore said, in DOSSO's case, the objective function is such that maximises the total price of consumption goods of the last period, which does not guarantee the production of all types of consumption goods. On the contrary, Kantorovich's planning model ensures that all types of consumption goods are produced.

In the next section, we try to carry out several simulations of China's economy with the framework of Kantorovich's planning.

## 9.3 Long-Term Planning Theory and China's Economy

This section uses integrated China's input-output tables which consist of six sectors for each year. The sectoral classification of fixed capital goods, current goods, and consumption goods is presented in Tables 9.1 and 9.2.

### *9.3.1 Kantorovich's Output Path*

First, it is assumed that no technical change occurred from 1995 to 1996 and from 1997 to 1999.

From $K$, $A$ and $F$ of only brand-new goods, the technical coefficient matrices *i.e.*, the rectangular input matrix $M$ and the rectangular output matrix $B$, that contain the aged fixed capital employed for the computation are counted backwards, as follows:

$$M_{96} = M_{95},\ M_{98} = M_{97},\ M_{99} = M_{97},$$
$$B_{96} = B_{95},\ B_{98} = B_{97},\ B_{99} = B_{97}.$$

---

[2]Even if the objective functions are different, the constraints in the planning period are substantively the same, so that the same results are drawn from the two models. This is an implication related to the strong turnpike theorem.

**Table 9.1** Classification of fixed capital and durability

| Sector | Items | Durability | Age distribution |
|---|---|---|---|
| 1 | Agriculture | 16 | 0, ..., 15 |
| 2 | Industry | 16 | 0, ..., 15 |
| 3 | Construction | 40 | 0, ..., 39 |
| 4 | Freight transportation and postal | 10 | 0, ..., 9 |
| 5 | Commerce and food service | 8 | 0, ..., 7 |

**Table 9.2** Classification of current goods and consumption goods

| Sector | Items |
|---|---|
| 1 | Agriculture |
| 2 | Industry |
| 3 | Construction |
| 4 | Freight transportation and postal |
| 5 | Commerce and food service |
| 6 | Non-substance production |

The 5-year plan from 1996 to 2000 is considered by making 1995 the initial year. Namely,

$$G = \begin{pmatrix} M_{96} & O & O & O & O & 0 \\ -B_{96} & M_{97} & O & O & O & 0 \\ O & -B_{97} & M_{98} & O & O & 0 \\ O & O & -B_{98} & M_{99} & O & 0 \\ O & O & O & -B_{99} & M_{00} & 0 \\ O & O & O & O & -B_{00} & \boldsymbol{a} \end{pmatrix}.$$

Let $d(0)$ be given by

$$\begin{aligned} d(0) = {}^t(&0.03348,\ \circ,\ 0.92096,\ \circ,\ 0.43310,\ \circ,\ 0.01034,\ \circ,\ 0.47924,\ \circ \\ &1.10666, 6.74796,\ 0.04973,\ 0.43646,\ 1.06864,\ 0.29998 \\ &0.05389,\ 0.07140,\ 0.02364,\ 0.05366,\ 0.02279,\ 0.03068), \end{aligned}$$

where $\circ$ is a row vector of zero, and the price unit of $d(0)$ is $10^{12}$ yuan. The initial conditions established herein show that the aged fixed capital at the initial time is ignored.

The product ratio of the request of the last period is carried out as follows:

$$\boldsymbol{a} = {}^t\big(0.5 \circ 0.5 \circ 0.5 \circ 0.5 \circ 0.5 \circ 0.75 \cdots 0.75\ 1 \cdots 1\big).$$

The above-mentioned linear programming problem is solved by using a five-period planning period. By taking the output ratio $q(t) = Bx(t)$, according to ages from the optimal solution, the results presented in Table 9.4 are obtained.[3]

The value of $Q^*$ in Table 9.4 shows that the ratio of the brand-new fixed capital goods, current goods, and consumption goods of the last period (2000) are produced by the desired ratio $\boldsymbol{a}$.

### 9.3.2 The Additional Restrictions on Fixed Capital

We consider the joint production system, in which aged fixed capital alone is jointly produced. We ignore that fixed capital is transferred between processes and sectors. Thus, with respect to fixed capital of age 0,

$$x_1(t) \geq x_2(t+1) \tag{9.3.1}$$

is held. Similarly, with respect to fixed capital of $i$ years old,

$$x_i(t) \geq x_{i+1}(t+1) \tag{9.3.2}$$

must be fulfilled. This is also the same for the production process of current and consumption goods. These constraints are termed as the fixed capital constraints.

The coefficient matrix $\mathbb{S}$ of the fixed capital constraint until the last period of the plan can be expressed as,

$$\mathbb{S} = \begin{pmatrix} -I & 0 & \cdots & 0 & I & & & & & 0 \\ & & -I & 0 & \cdots & 0 & I & & & 0 \\ & & & & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ & & & & & -I & 0 & \cdots & 0 \; I & 0 \end{pmatrix}. \tag{9.3.3}$$

Therefore, the linear programming problem including the fixed capital constraint is as follows:

$$\max \left\{ \boldsymbol{v}\boldsymbol{x}(t) \,\middle|\, \begin{pmatrix} G \\ \mathbb{S} \end{pmatrix} \boldsymbol{x}(t) \leq \begin{pmatrix} \boldsymbol{d} \\ 0 \end{pmatrix}, \boldsymbol{x}(t) \geq 0 \right\}. \tag{9.3.4}$$

The dual problem of (9.3.4) is as follows:

$$\min \left\{ \boldsymbol{y}(t) \begin{pmatrix} \boldsymbol{d} \\ 0 \end{pmatrix} \,\middle|\, \boldsymbol{y}(t) \begin{pmatrix} G \\ \mathbb{S} \end{pmatrix} \geq \boldsymbol{v}, \boldsymbol{y}(t) \geq \circ \right\}. \tag{9.3.5}$$

[3] See the optimal value of $Q^*$. In addition, the statement of aged fixed capital is omitted.

Based on the above fixed capital constraints, the calculation result $Q^*_K$ of the output classified by type is presented in Table 9.4.

We see that there is no large error in terms of the output ratio compared with the calculation result that does not add the fixed capital constraint from Table 9.4. For the output itself, however, it turns out that some reductions are brought about.

### 9.3.3 The Long-Term Plan with a Labour Resource Constraint

This subsection examines the case in which the time shift of the amount of labour resources becomes a constraint.

The amount $N(t)$ of labour supply at period $t$ is thus given. Since labour necessary for period $t$ is $Lx(t)$,

$$Lx(t) \leq N(t) \tag{9.3.6}$$

represents the labour resource restriction. The labour coefficient in each year is assumed to be as follows:

$$L_{96} = L_{95},\ L_{98} = L_{97},\ L_{99} = L_{97}.$$

The above-mentioned conditions are expressed until the last period of a plan in order to obtain

$$\begin{pmatrix} L_{96} & \ominus & \ominus & \ominus & \ominus & 0 \\ \ominus & L_{97} & \ominus & \ominus & \ominus & 0 \\ \ominus & \ominus & L_{98} & \ominus & \ominus & 0 \\ \ominus & \ominus & \ominus & L_{99} & \ominus & 0 \\ \ominus & \ominus & \ominus & \ominus & L_{00} & 0 \end{pmatrix} \begin{pmatrix} x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \\ k \end{pmatrix} \leq \begin{pmatrix} N(1) \\ N(2) \\ N(3) \\ N(4) \\ N(5) \end{pmatrix}. \tag{9.3.7}$$

Here, the coefficient parts will be denoted by

$$\mathbb{H} = \begin{pmatrix} L_{96} & \ominus & \ominus & \ominus & \ominus & 0 \\ \ominus & L_{97} & \ominus & \ominus & \ominus & 0 \\ \ominus & \ominus & L_{98} & \ominus & \ominus & 0 \\ \ominus & \ominus & \ominus & L_{99} & \ominus & 0 \\ \ominus & \ominus & \ominus & \ominus & L_{00} & 0 \end{pmatrix},\ \mathbb{N}(t) = \begin{pmatrix} N(1) \\ N(2) \\ N(3) \\ N(4) \\ N(5) \end{pmatrix},$$

and this leads to the linear programming problem with the labour resource constraint.

**Table 9.3** The optimal path of necessary labour and the given amount of labour supply

| Year | 1995 | 1996 | 1997 | 1998 | 1999 | 2000 |
|---|---|---|---|---|---|---|
| $Lx(t)$ | 693124.09 | 428214.52 | 429759.06 | 407444.83 | 553001.63 | 728129.45 |
| $\mathbb{N}(t)$ | 693124.09 | 700085.47 | 706855.76 | 713290.91 | 720366.85 | 728129.45 |

The source of labour supply $\mathbb{N}$: LABORSTA (http://laboursta.ilo.org) Unit: 1000 people

**Table 9.4** Actual values and the optimal solution by Kantorovich's long-term planning: 0-age goods

| Sector | | 1995 | 1997 | | | | 2000 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $Q_0$ | Actual | $Q^*$ | $Q_K^*$ | $Q_L^*$ | Actual | $Q^*$ | $Q_K^*$ | $Q_L^*$ |
| Fixed capital | 1 | 0.03348 | 0.00041 | 0.23308 | 0.23300 | 0.00303 | 0.02994 | 0.54797 | 0.54778 | 0.00449 |
| | 2 | 0.92096 | 1.51673 | 1.78096 | 1.78033 | 0.02262 | 1.43884 | 0.54797 | 0.54778 | 0.00449 |
| | 3 | 0.43310 | 0.40455 | 0.01767 | 0.01647 | 0.00410 | 1.32926 | 0.54797 | 0.54778 | 0.00449 |
| | 4 | 0.01034 | 0.00096 | 0.08394 | 0.08537 | 0.00096 | 0.01998 | 0.54797 | 0.54778 | 0.00449 |
| | 5 | 0.47924 | 0.04474 | 0.27718 | 0.27708 | 0.00348 | 0.91835 | 0.54797 | 0.54778 | 0.00449 |
| Current goods | 1 | 1.10666 | 1.34108 | 0.34963 | 0.34950 | 0.00455 | 1.39858 | 0.82195 | 0.82167 | 0.00673 |
| | 2 | 6.74796 | 8.68378 | 2.67144 | 2.67050 | 0.03393 | 11.7721 | 0.82195 | 0.82167 | 0.00673 |
| | 3 | 0.04973 | 0.10286 | 0.02650 | 0.02471 | 0.00615 | 0.13626 | 0.82195 | 0.82167 | 0.00673 |
| | 4 | 0.43646 | 0.39853 | 0.12591 | 0.12806 | 0.00144 | 0.54592 | 0.82195 | 0.82167 | 0.00673 |
| | 5 | 1.06864 | 1.43181 | 0.41577 | 0.41562 | 0.00523 | 1.99904 | 0.82195 | 0.82167 | 0.00673 |
| | 6 | 0.29998 | 0.45482 | 0.15712 | 0.15807 | 0.00147 | 0.67123 | 0.82195 | 0.82167 | 0.00673 |
| Consumption goods | 1 | 0.05389 | 0.07064 | 0.46617 | 0.46600 | 0.00606 | 0.07995 | 1.09594 | 1.09555 | 0.00897 |
| | 2 | 0.07140 | 0.07560 | 3.56191 | 3.56066 | 0.04524 | 0.06603 | 1.09594 | 1.09555 | 0.00897 |
| | 3 | 0.02364 | 0.01929 | 0.03534 | 0.03295 | 0.00820 | 0.02225 | 1.09594 | 1.09555 | 0.00897 |
| | 4 | 0.05366 | 0.06174 | 0.16788 | 0.17074 | 0.00192 | 0.08031 | 1.09594 | 1.09555 | 0.00897 |
| | 5 | 0.02279 | 0.02294 | 0.55436 | 0.55416 | 0.00697 | 0.02858 | 1.09594 | 1.09555 | 0.00897 |
| | 6 | 0.03068 | 0.06257 | 0.20950 | 0.21076 | 0.00196 | 0.20405 | 1.09594 | 1.09555 | 0.00897 |

Note: $Q^*$, $Q_K^*$ (fixed capital constraint), $Q_L^*$ (labour resource constraint) express the optimal solution (brand-new goods) Unit: $10^{12}$ Yuan

That is,

$$\max \left\{ \boldsymbol{v}\boldsymbol{x}(t) \;\middle|\; \begin{pmatrix} G \\ \mathbb{H} \end{pmatrix} \boldsymbol{x}(t) \le \begin{pmatrix} \boldsymbol{d} \\ \mathbb{N}(t) \end{pmatrix}, \boldsymbol{x}(t) \ge 0 \right\}. \tag{9.3.8}$$

The Kantorovich-type output time series, which are classified by type in terms of the labour resource constraint, are shown by $Q_L^*$ in Table 9.4.

The optimal path in the long-term planning and the given amount of labour supplies under labour resource constraint are presented in Table 9.3.

The optimal solution (only brand-new goods) in terms of actual value and the long-run plan by year are shown in Table 9.4 and Figs. 9.1, 9.2 and 9.3.

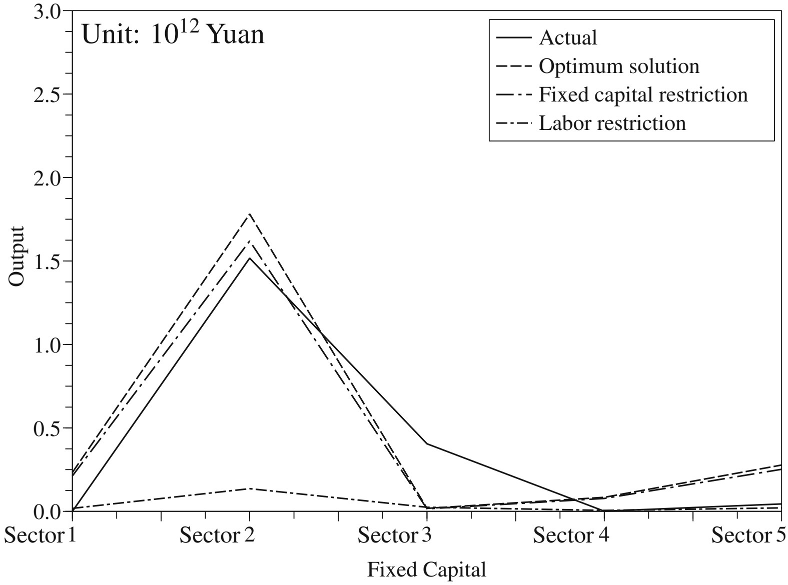


**Fig. 9.1** The optimal solution of Kantorovich's long-term planning and actual values (1997, fixed capital)

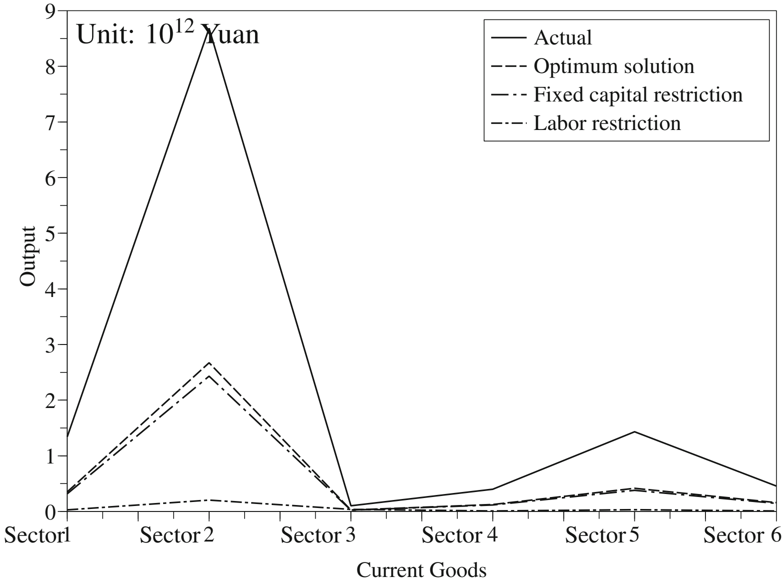


**Fig. 9.2** The optimal solution of Kantorovich's long-term planning and actual values (1997, current goods)

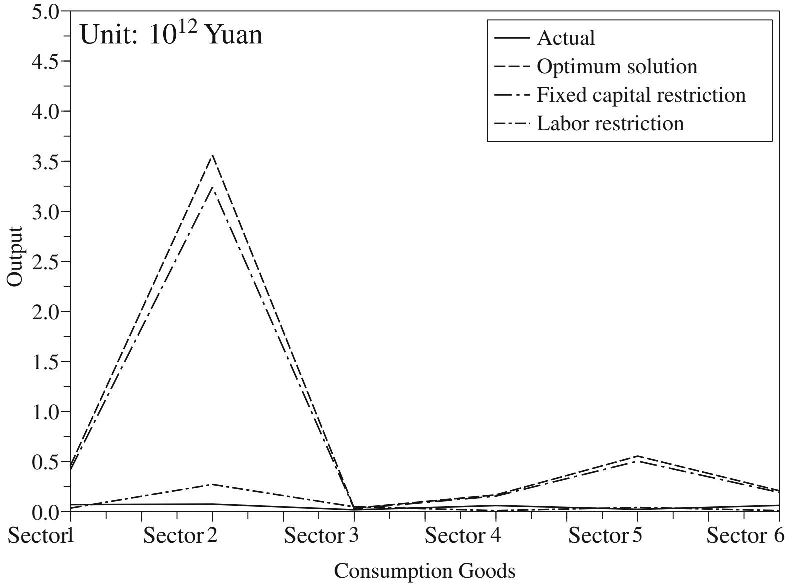


**Fig. 9.3** The optimal solution of Kantorovich's long-term planning and actual values (1997, consumption goods)

## 9.4 Concluding Remarks

This chapter applied the framework of the Marx-Sraffa-von Neumann model to China's economy. First, we applied Kantorovich's long-term planning theory, which states that the production of all consumption goods is guaranteed and exports and imports are ignored, to China's economy. We made the following three findings.

First, from the calculation result of fixed capital shown in Fig. 9.1, the actual value of other sectors seems to approach the theoretical value except sector 3 (construction sector), which is due to that political factors rather than the planned target were influencial than in other sectors. Likewise, the actual value of the current goods sector is above the theoretical value (Fig. 9.2).

Second, although the actual values of sectors 3, 4 and 6 are approaching the theoretical value, as shown by the calculation result of the consumption goods sector in Fig. 9.3, sectors 1, 2 and 5 are much lower than the theoretical value.

Third, as shown in Table 9.4, when the binding condition of fixed capital is added, production quantity tends to fall. By contrast, when the labour necessary for the plan of the last period and the actual amount of labour supply are equal, full employment is attained as production quantity falls under the conditions of the labour resource constraint.

In China, an infrastructural improvement plan has continued actively since 1990s under the control of the government. As shown from the simulation of the turnpike, investment of fixed capital seems to be managed properly in general.

Concerning current or consumption goods, the socialistic market economy has been prominent, especially since the 1990s, as shown by the turnpike simulation, Thus, there has been a certain amount of deviation in actual value from the optimal path.

Therefore, it seems that computing turnpikes will be helpful for the macro control of the actual economy. Thus, this can be used for economic policy-making, as a tool for the objective analysis of the socialist economy when an economic plan clearly exists.

# Chapter 10
# Concluding Remarks

**Summary of Analyses**. Let us summarise major contents of this small book.

As shown by the title, theory and its application are presented in this book.

We focussed on the structural and dynamic property of fixed capital.

In the discussion of shaping SON, it is shown that elimination of prices of aged fixed capital is equivalent to introducing the rate of depreciation. In the original joint-production system, the rate of profit and the prices of aged fixed capital is mutually related in a simple manner, while in SON the relationship appears rather complicated.

This is, however, enough to suggest to economists that heavy influence will be spread on the commodity production system from fixed capital. Renewal dynamics casts a big shadow on dynamics of economies. Even though cycles generated by MERLKS effect is converging, its converging velocity is relatively small, *i.e.*, they converge very slowly, spikes created by the cycle will certainly give large disturbing effects on the economy. Not only from the angle of accounting, but also from the viewpoint of economics, we can confirm that fixed capital itself is a major source of disequilibrium of the commodity production system.

If we start from the generic joint-production system and try to find economic equilibria from the eigensystem of the system, it is seen that equilibria are usually unstable. This implies that the commodity production system has, in general, unstable equilibria of production prices and quantities, or distribution and growth.

Fixed capital, as a materialistic basis of production, plays important roles in the economy. Data of fixed capital, however, is very limited, so that positive analysis of the economy with fixed capital is hard to carry out. We estimated marginal fixed capital coefficients of China's economy on the basis of the linear economic model *à la* Marx-Sraffa. We showed that such coefficients can be obtained from data of gross investment, which can be grasped as flows.

By applying estimated fixed-capital coefficients, we computed labour values and the turnpike. From the labour values and production prices of China's economy, it is seen that agriculture is facing some structural difficulties, in view of Marx's proposition on the transformation problem.



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By simulating the turnpike path of China's economy, it is seen that China's economy had been running very close to the turnpike, and hence the performance was good, although the economy seemed to be a little overheated, namely, the consumption side is a little backward.

**Future of Studies**. In order to facilitate studies of the economy with fixed capital, the first requirement is data of fixed capital. It might be difficult to compile data of fixed capital, but it is the most desirable matter.

We did not discuss the problem of durable consumers' goods. Approaches to fixed capital will be applied to this.

In this book, we assumed that labour is homogeneous. The problem of investment in education and the accumulation of human capital, or skilled labour power, remains untouched. This field of study is also a possible arena to that the approach to fixed capital will be applied.

It is quiet certain that, with appropriate data, Marx's political economy can tell us difficulties of the economy, in which we live.

# Appendix A
# Input-Output Tables and Marx's 2-Sector Model

## A.1 Introduction

In this appendix, we discuss how to build Marx's 2-sector model of production and consumption from the input-output tables of China, and then simulate the turnpike of China's economy. Fixed capital is not the main point in this appendix, for fixed capital data are limited in the input-output data.

First, we explain some features of China's input-output tables.

Next, we begin with preprocessing input-output data. Framework of analysis in this appendix is inherited from Fujimori (1992a).

Third, we construct Marx's 2-sector models of China's economy of the year concerned, *i.e.*, 1981–2007. With these Marx's 2-sector models, we can evaluate important economic variables, such as the rates of profit, surplus value, growth, accumulation *etc.*

Lastly, we compute the turnpike for China's economy. We also plot the consumption-investment curves for China's economy.

**China's input-output tables**. It will be appropriate to describe some notes about data of China's input-output data and others at the outset.

The inter-industry relations data uses the National Bureau of Statistics of China data for the years 1981 (24 sectors), 1987, 1990, 1992, 1995 (33 sectors), 1997, 2000 (40 sectors), 2002, 2005, and 2007 (42 sectors). The source for the labour data comes from the *International Labor Organization* (ILO) *Yearbook of Labor Statistics* (Japanese edition), for the year concerned.

The input-output table data system was first introduced into China in the 1950s and 1960s. The objective of research on input-output tables was the analysis and application of input-output methods. In 1974, government agencies such as the National Bureau of Statistics, the National Planning Committee, and the Chinese Academy of Sciences compiled the first input-output table in 1973. This table included 61 types of real commodities. In 1982 the National Bureau of Statistics and the National Planning Committee compiled the 1981 input-output table (prototype table) from the Material Product System (MPS, the actual model). After this, the 1983 nationwide



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MPS was also compiled, and, in 1987, the State Council officially defined criteria for input-output surveys and the system for their compilation. Since then, tables for the years of 1987, 1990, 1992, 1995, 1997, 2000 and 2002 have been compiled based on the System of National Account (SNA). The basic conceptual structure of China's input-output tables are described in Centre of Economic Forecasting (1987).[1]

Regarding the names of the sectors taken up in this chapter refer to the inter-industry table of each year. Among the value-added entries, employees' wages appear as wages, whereas manufacturing taxes plus business surplus do as profits. As for the 24-sector table, also refer to Chap. 7, p. 66 and Chap. 8, p. 80.

## A.2 Preprocessing Input-Output Tables

### A.2.1 Computing the Coefficients for Input-Output Systems

We carry out the calculations in the following order.

(i) The input matrix, labour vector, wage-good bundle and accumulation ratio are obtained from official statistical data.
First, the input coefficient $A^*$ is computed by

$$A^* = \left(\frac{X_{ij}}{X_j}\right).$$

Overall working hours $H$ and overall value added $V$ are given respectively by

$$H = Nh, \tag{A.2.1}$$

$$V = \sum_{i=1}^{n} (\mathcal{W}_i + \mathcal{U}_i + \mathcal{T}_i). \tag{A.2.2}$$

The labour input coefficient for each sector is determined by

$$L^* = ((\mathcal{W}_{1j} + \mathcal{U}_{1j} + \mathcal{T}_{1j})H/V)/X_j. \tag{A.2.3}$$

The per capita wage goods bundle $f^*$ is treated as equal to the consumption entry divided by the overall workforce, so that the wage goods vector per unit of labour is

$$f^* = ((C_i^1 + C_i^2 + C_i^3)/N)/h. \tag{A.2.4}$$

Since total profit $S^*$ is evaluated as $S^* = Y - W^*$, the accumulation ratio $\alpha^*$ is determined by

[1] As for detailed methods of compiling China's input-output tables, one recent work is Qi (2003).

$$\alpha^* = \frac{K^*}{S^*}. \tag{A.2.5}$$

(ii) $c^*$ should be determined in such a way that $\lambda_M = 1$ holds. That is, $L^*$ and $f^*$ are given by (A.2.3) and (A.2.4) respectively. Coefficient matrix $M$ is expressed by

$$M = A^* + c^* f^* L^*. \tag{A.2.6}$$

As a result, $c^*$ can be solved so that $\lambda_M = 1$. Incidentally, because $g_c = 0$, this is the point at which wages are at their highest.

(iii) When $c^*$is determined, this fixes $F^* = c^* f^*$. $F^*$ is the basket of wage goods in case of the profit rate being zero. We write

$$M^* = A + \vartheta F^* L^*. \tag{A.2.7}$$

$\vartheta$ runs in the interval of $0 < \vartheta < 1$, and $M^*$ depends on $\vartheta$. Thus, one obtains

$$g_c = \frac{1}{\lambda_{M^*}} - 1.$$

$\vartheta$ reflects real wages, and $(\vartheta, g_c)$ shows the consumption-investment curve. If $\vartheta = 0$, then $g_c = \frac{1}{\lambda_A} - 1$, corresponding to the maximum rate of profit $R$.

(iv) $g_c^*$ is found, so that one can examine $g^* = \alpha^* r = \alpha^* g_c$. $g_c^*$ is the von Neumann growth rate that corresponds to the real economy. so $(\vartheta^*, g_c^*)$, point $E$, is below the consumption-investment curve.

Note that the accumulation rate will not reach 1, because unproductive consumption exists in the real economy.

China's statistical base data is shown in Table A.1, and the result of the calculation of various indices is shown in Table A.2. Furthermore, the consumption-investment

**Table A.1** Major data

| | $Y$ (B.Y.) | $W^*$ (B.Y.) | $N$ (10$K$) | $K^*$ (B.Y.) | $h$ (Hours) | $g^*$ (%) |
|---|---|---|---|---|---|---|
| 1981 | 4901.4 | 820.0 | 43725 | 1581.0 | 1937 | 5.2 |
| 1987 | 11784.7 | 1881.1 | 52783 | 4322.0 | 1908 | 9.4 |
| 1990 | 18319.5 | 2951.1 | 64749 | 6444.0 | 1895 | 5.0 |
| 1992 | 25863.7 | 3939.2 | 66152 | 9636.0 | 2225 | 12.8 |
| 1995 | 58510.5 | 8100.0 | 68065 | 23877.0 | 2057 | 10.2 |
| 1997 | 74894.2 | 9405.3 | 69820 | 28457.6 | 1895 | 8.8 |
| 2000 | 89340.9 | 10656.2 | 72085 | 32499.8 | 1800 | 8.0 |
| 2002 | 107897.6 | 13161.1 | 73740 | 42304.9 | 1900 | 8.0 |
| 2005 | 184937.4 | 20627.1 | 75825 | 88773.6 | 2390 | 10.4 |
| 2007 | 265810.3 | 29471.5 | 76990 | 137323.9 | 2275 | 11.4 |

*Note* $h$ of 1981, 1985, 1987 and 2002 are estimation

**Table A.2** Estimated values

| | $V$ (B.Y.) | $S^*$ (B.Y.) | $\alpha^*$ (%) | $g_c$ (%) | $c^*$ | $\vartheta^*$ | $R$(%) |
|---|---|---|---|---|---|---|---|
| 1981 | 3940.206 | 4081.4 | 38.7 | 13.4 | 1.537 | 0.770 | 97.6 |
| 1987 | 9043.175 | 9903.6 | 43.6 | 21.5 | 1.554 | 0.632 | 82.0 |
| 1990 | 13258.518 | 15368.4 | 41.9 | 11.9 | 1.610 | 0.766 | 66.8 |
| 1992 | 23106.907 | 21924.5 | 44.0 | 29.1 | 1.605 | 0.446 | 60.2 |
| 1995 | 51852.328 | 50410.5 | 47.4 | 21.5 | 1.697 | 0.559 | 58.2 |
| 1997 | 65391.852 | 65488.9 | 43.5 | 20.3 | 1.702 | 0.583 | 58.3 |
| 2000 | 77741.367 | 78684.7 | 41.3 | 19.4 | 1.639 | 0.580 | 53.9 |
| 2002 | 103118.337 | 94736.5 | 44.7 | 17.9 | 1.702 | 0.639 | 60.6 |
| 2005 | 157084.977 | 164310.3 | 54.0 | 19.2 | 1.900 | 0.556 | 47.4 |
| 2007 | 228788.278 | 236338.8 | 58.1 | 19.6 | 2.026 | 0.543 | 46.2 |

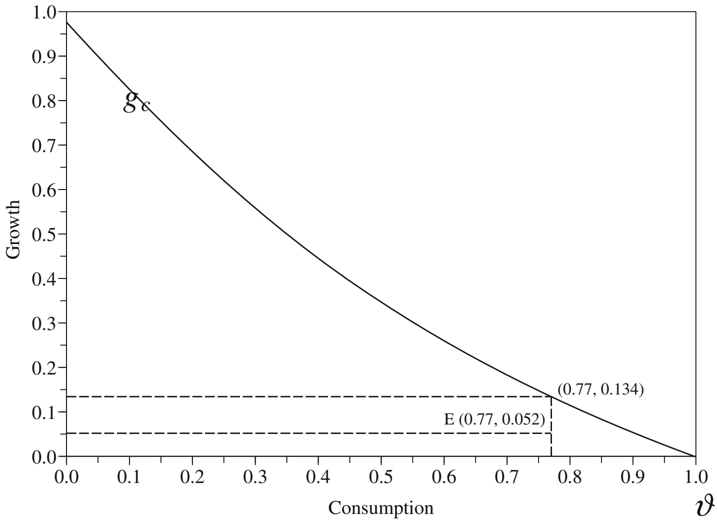


**Fig. A.1** Consumption-growth curve of 1981

curve calculated for every year is shown by each of Figs. A.1, A.2, A.3, A.4, A.5, A.6, A.7, A.8, A.9 and A.10.

According to the results of calculation above, it is clear that point $E$ is disconnected from the consumption-investment curve expressed as $(\vartheta, g_c)$. At point $E$ the growth rate in the real economy of every year is less than every theoretical value on the consumption-investment curve.

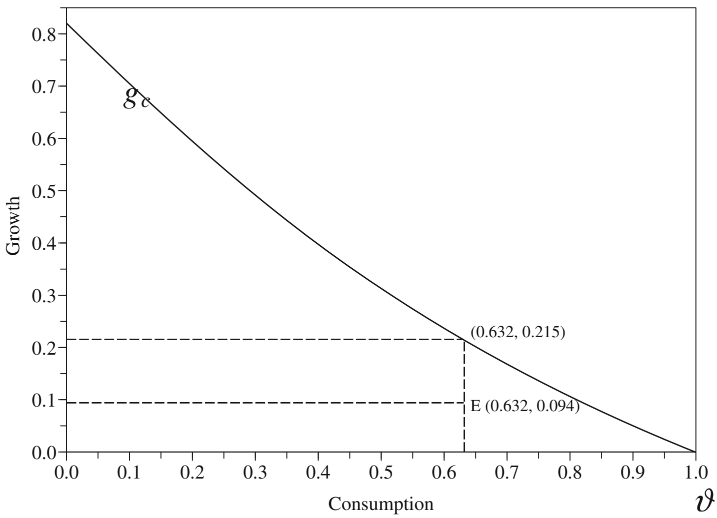


**Fig. A.2** C-G curve of 1987

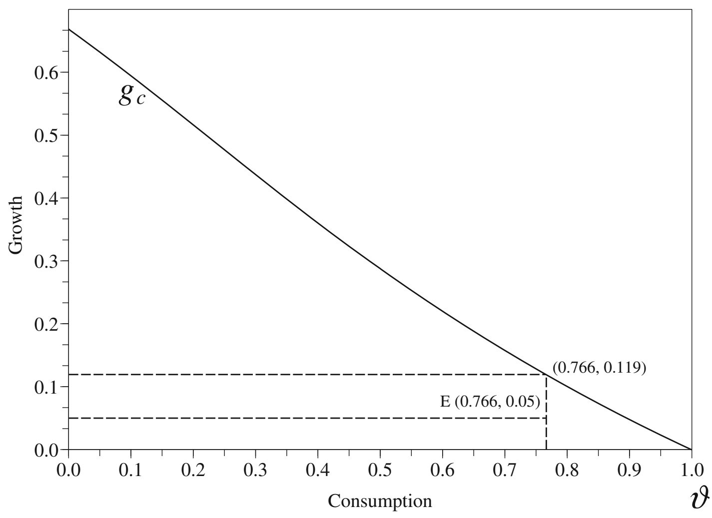


**Fig. A.3** C-G curve of 1990

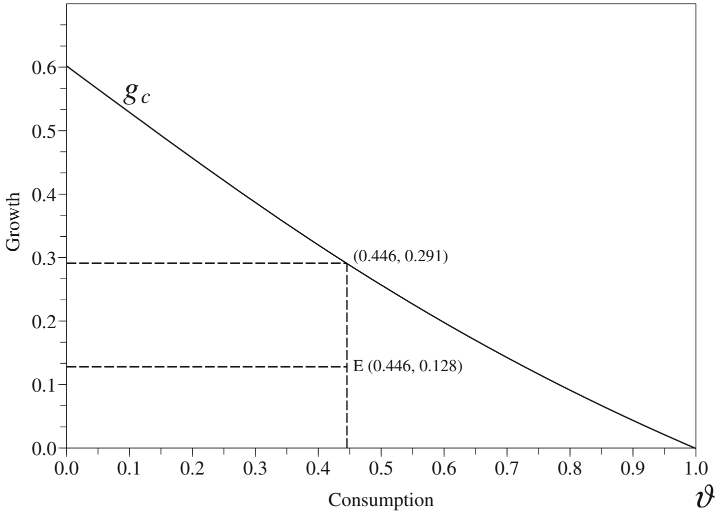


**Fig. A.4** C-G curve of 1992

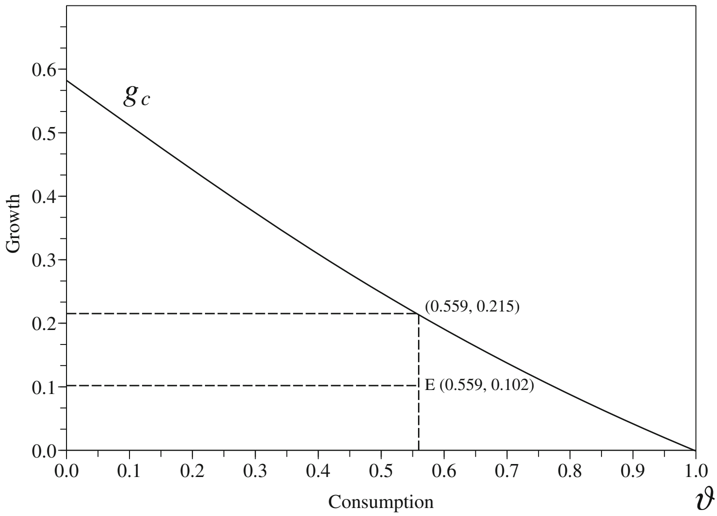


**Fig. A.5** C-G curve of 1995

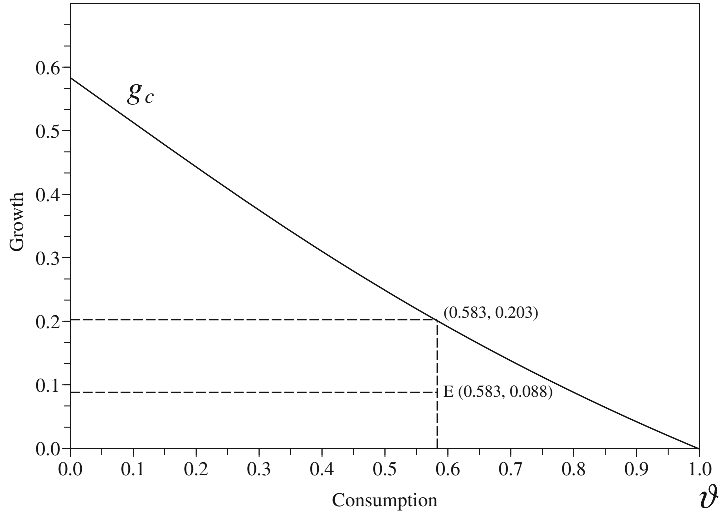


**Fig. A.6** C-G curve of 1997

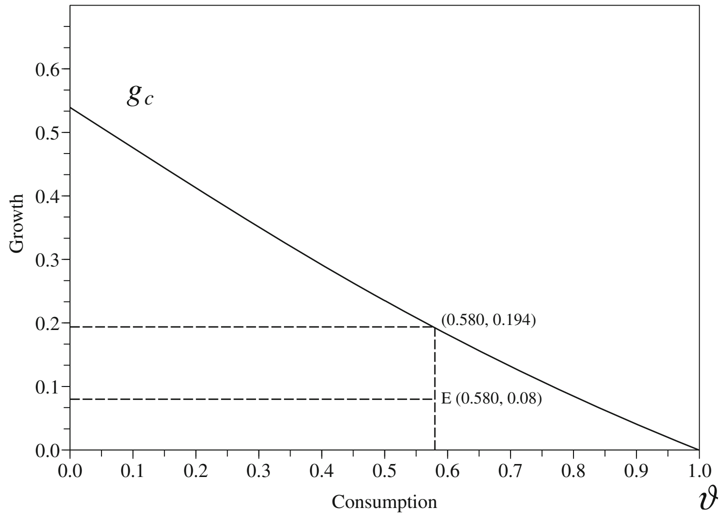


**Fig. A.7** C-G curve of 2000

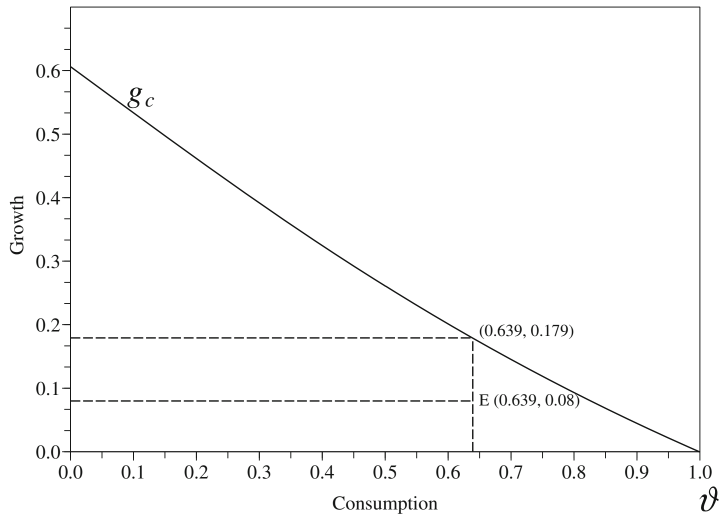


**Fig. A.8** C-G curve of 2002

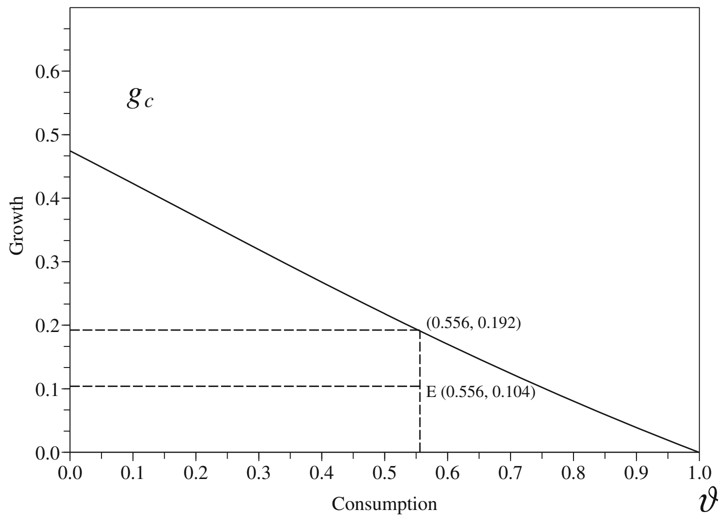


**Fig. A.9** C-G curve of 2005

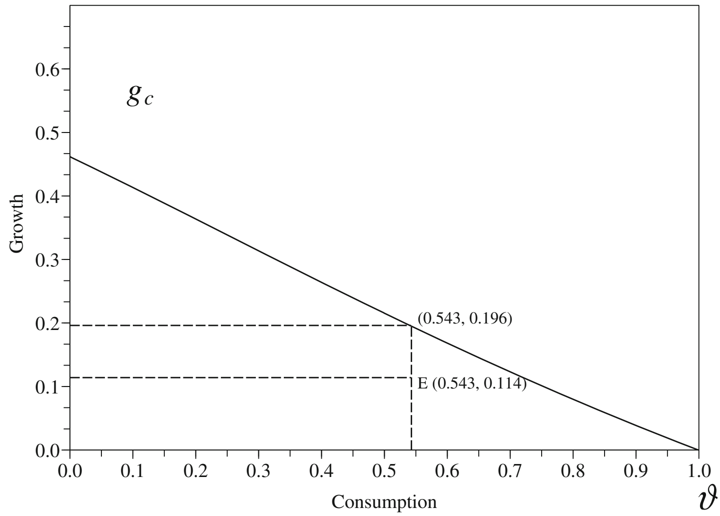


**Fig. A.10** C-G curve of 2007

## A.3 Marx's 2-Sector Model

Marx's 2-sector model is made up of sector I, capital goods production, and sector II, consumer goods production. It reflects the most fundamental economic structures, such as the ratio of labour to capital and the ratio of wages to profits. We construct China's 2-sector economic model here from the input-output tables.

### *A.3.1 The Ratio $\lambda_i$ of Capital Goods to Demand in a Closed Economy*

First, if we put that $x^K$ and $x^C$ stand for respectively outputs in capital sector and consumption sector, then the capital goods to demand ratio in a closed economy is defined as follows.

$$\lambda_i = \frac{x_i^K}{x_i}. \tag{A.3.1}$$

From this definition, in production per unit of industry $i$, $\lambda_i$ shows the capital sectors production, and $1 - \lambda_i$ shows the consumer sectors production.

Using $\lambda_i$, in the two sectors capital $k_i$, wages $W_i$, profit $\Pi_i$, and total production $Y_i$ are expressed as follows:

$$k_I = \sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_j a_{ij} x_j, \tag{A.3.2}$$

$$k_{II} = \sum_{i=1}^{n}\sum_{j=1}^{n} (1-\lambda_j) a_{ij} x_j, \tag{A.3.3}$$

$$W_I = \sum_{i=1}^{n} \lambda_i w_i, \tag{A.3.4}$$

$$W_{II} = \sum_{i=1}^{n} (1-\lambda_i) w_i, \tag{A.3.5}$$

$$\Pi_I = \sum_{i=1}^{n} \lambda_i s_i, \tag{A.3.6}$$

$$\Pi_{II} = \sum_{i=1}^{n} (1-\lambda_i) s_i, \tag{A.3.7}$$

$$Y_I = \sum_{i=1}^{n} \lambda_i x_i, \tag{A.3.8}$$

$$Y_{II} = \sum_{i=1}^{n} (1-\lambda_i) x_i. \tag{A.3.9}$$

Again, it is clear from investment vector $\Delta k_i$ and consumption vector $C$ that total investment $\mathcal{K}$ and total consumption $\mathcal{C}$ amount to

$$\mathcal{K} = \sum_{i=1}^{n} \Delta k_i, \tag{A.3.10}$$

$$\mathcal{C} = \sum_{i=1}^{n} C_i. \tag{A.3.11}$$

### *A.3.2 Difference in Data: Major Renewal*

In this subsection, we can consider two kinds of processes for the calculation of depreciation entries based on differences in the data of China's input-output tables.

First, dummy industries (imaginary sectors) are created and inserted into the intermediate-demand entries, for "major renewal" entry was added into the final demand column in 1981 Chinese input-output table. That is, the industry $n+1$ is assumed to be a dummy industry concerning fixed capital.

The input-output relationship included in a dummy sector will be expressed as follows:

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \\ x_{n+1}^d \end{pmatrix} = \left(\begin{array}{ccc|c} x_{11} & \dots & x_{1n} & \sum_{j=1}^{n} r_{1,j} \\ \vdots & \ddots & \vdots & \vdots \\ x_{n1} & \dots & x_{nn} & \sum_{j=1}^{n} r_{n,j} \\ \hline \sum_{i=1}^{n} d_{i,1} & \dots & \sum_{i=1}^{n} d_{i,n} & 0 \end{array}\right) + \begin{pmatrix} C_1 \\ \vdots \\ C_n \\ 0 \end{pmatrix} + \begin{pmatrix} \Delta K_1 \\ \vdots \\ \Delta K_n \\ 0 \end{pmatrix}.$$

Next, in case that "major renewal" entry disappeared in the data after 1987, we process them as follows.

We can regard the investment on fixed capital as part of physical investment for production. For convenience, investment including the renewal of fixed capital is often referred to as "gross investment." In the real economic system, renewal $r_{ij}$ and depreciation $d_{ij}$ of fixed capital do not necessarily match. However, if the economy is in equilibrium, we can assume that their relationship is given by

$$r_{ij} = d_{ij}.$$

Consequently, depreciation $D_i$ of sector $i$ amounts to

$$D_I = \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_j r_{ij} = \sum_{j=1}^{n} \lambda_j \left( \sum_{i=1}^{n} d_{ij} \right), \tag{A.3.12}$$

$$D_{II} = \sum_{i=1}^{n} \sum_{j=1}^{n} (1 - \lambda_j) r_{ij} = \sum_{j=1}^{n} (1 - \lambda_j) \left( \sum_{i=1}^{n} d_{ij} \right). \tag{A.3.13}$$

Here the total consumed capital in industry $i$, that is $k_i^\dagger$ of sector $i$, where $i = I, II$, amounts to

$$k_i^\dagger = k_i + D_i. \tag{A.3.14}$$

From the viewpoint of investment (the vertical sum of the input-output table), we have

$$Y_I = k_I^\dagger + W_I + \Pi_I, \tag{A.3.15}$$

$$Y_{II} = k_{II}^\dagger + W_{II} + \Pi_{II}. \tag{A.3.16}$$

Denoting the net investment of fixed capital vector by $\Delta F$, we obtain, from the composition of gross investment in this input-output system,

$$\Delta K = \Delta F + \Delta k \tag{A.3.17}$$

in the total net investment vector $\Delta K$. However, net investment on fixed capital $\mathcal{K}^{\dagger}$ amounts to

$$\mathcal{K}^{\dagger} = \sum_{i=1}^{n} \Delta F_i. \tag{A.3.18}$$

### A.3.3 The 2-Sector Model in an Open Economy

In case there are international transactions, it is necessary to revise the ratio $\lambda_i$, for gross production is generally not equal to domestic aggregate demand. The equilibrium equation in an open economy is as follows. Let $E$ and $M$ denote exports and imports, respectively.

$$x = A^{\dagger}x + C + \Delta K + E - M. \tag{A.3.19}$$

Here, domestic aggregate demand $H_i$ amounts to

$$H_i = \sum_{j=1}^{n} a_{ij}x_j + \Delta K_i + C_i. \tag{A.3.20}$$

In an open economy model which includes international transactions, ratio $\lambda_i$ can be redefined as

$$\lambda_i = 1 - \frac{C_i}{H_i}. \tag{A.3.21}$$

On the other hand, in an open economy $\mathcal{E}_I$ and $\mathcal{E}_{II}$ satisfy

$$\sum_{j=1}^{n} \lambda_j x_j = \sum_{i=1}^{n}\Big(\sum_{j=1}^{n} x_{ij} + \Delta K_i\Big) + \mathcal{E}_I \tag{A.3.22}$$

$$\sum_{j=1}^{n}(1-\lambda_j)x_j = \sum_{j=1}^{n} C_j + \mathcal{E}_{II}. \tag{A.3.23}$$

Here, net-exports = exports − imports.

In a 2-sector open economy the horizontal equilibrium equation amounts to

**Table A.3** 2-sec open economy

| | I | II | Final demand | Net exports | Total |
|---|---|---|---|---|---|
| I | $k_I^\dagger$ | $k_{II}^\dagger$ | $\mathcal{K} + \mathcal{K}^\dagger$ | $\mathcal{E}_I$ | $Y_I$ |
| II | | | $\mathcal{C}$ | $\mathcal{E}_{II}$ | $Y_{II}$ |
| Wages | $W_I$ | $W_{II}$ | | | |
| Profits | $\Pi_I$ | $\Pi_{II}$ | | | |
| Total | $Y_I$ | $Y_{II}$ | | | |

$$Y_I = k_I^\dagger + k_{II}^\dagger + \mathcal{K} + \mathcal{K}^\dagger + \mathcal{E}_I, \tag{A.3.24}$$

$$Y_{II} = \mathcal{C} + \mathcal{E}_{II}. \tag{A.3.25}$$

Based on the analysis above, the 2-sector table for an open economy can be recompiled as Table A.3.

## A.3.4 China's 2-Sector Input-Output Tables

In the process shown above China's 2-sector input-output table compiled from the multi-sector one is shown in Table A.4.

Using the figures from Table A.4 the calculation results of composition and distribution indices are as shown in Table A.5. Here, $\kappa$, $\mu$, $\pi$, $\zeta$, $\delta$ and $\sigma$ denote the organic composition of capital, the rate of surplus value, the rate of profit, the output ratio of production, the rate of growth and the accumulation-profit ratio, respectively, with subscripts indicating the sector concerned.

From these structural composition and distribution indices, we can arrive at the following:

(1) Sector ratio $\zeta > 1$. That is, the weight of capital goods is greater than that of consumption in the economy.
(2) $\kappa_I > \kappa_{II}$, $\mu_I > \mu_{II}$. In 1987, 1990, 1992 and 2000 $\pi_I > \pi_{II}$, and in 1981, 1995, 1997, 2002, 2005 and 2007 $\pi_I < \pi_{II}$.
(3) The rate of profit in both sectors tends to decline. This is because year-by-year fixed capital investments have increased.
(4) The surplus value ratio of both sectors tended to gradually decrease until 2000, while tended to gradually increase after 2000.
(5) The capital growth rate gradually decreased until the latter half of the 1980s, while from the early 1990s to the present day there has been a strong evidence showed the trend towards gradual increase.
(6) Since 1987 the accumulation to profit ratio $\sigma$ has been considerably high. This is because profits invested on government expenditure has been decreased.

**Table A.4** 2-sector input-output table (B.Y.)

| | I | II | Final demand | Net export | Total |
|---|---|---|---|---|---|
| I | 3821 (322) | 1288 (89) | 891 | 455 | 6454 |
| II | | | 2584 | 10 | 2594 |
| Wages | 1279 | 738 | | | |
| Profits | 1355 | 568 | | | |
| 1981 total | 6454 | 2594 | | | 9048 |
| I | 12277 (896) | 3163 (306) | 3979 | −55 | 19364 |
| II | | | 6466 | −167 | 6299 |
| Wages | 3536 | 2145 | | | |
| Profits | 3551 | 990 | | | |
| 1987 total | 19364 | 6299 | | | 25663 |
| I<br>II | 21658 (1463) | 4983 (499) | 5579<br>9509 | 622<br>−139 | 32843<br>9371 |
| Wages | 5717 | 2949 | | | |
| Profits | 5468 | 1438 | | | |
| 1990 total | 32843 | 9371 | | | 42213 |
| I | 37802 (2728) | 7555 (810) | 8503 | 535 | 54396 |
| II | | | 14187 | −118 | 14068 |
| Wages | 7757 | 4296 | | | |
| Profits | 8837 | 2217 | | | |
| 1992 total | 54396 | 14068 | | | 68464 |
| I | 87643 (5936) | 17050 (1660) | 20485 | 1123 | 126301 |
| II | | | 30824 | −580 | 30244 |
| Wages | 19770 | 8124 | | | |
| Profits | 18888 | 5070 | | | |
| 1995 total | 126301 | 30244 | | | 156545 |
| I | 112785 (7992) | 21667 (2320) | 23851 | 3298 | 161601 |
| II | | | 38799 | −556 | 38243 |
| Wages | 29333 | 12207 | | | |
| Profits | 19483 | 4369 | | | |
| 1997 total | 161601 | 38243 | | | 199844 |
| I | 152050 (11479) | 27762 (3127) | 25241 | 4677 | 209730 |
| II | | | 48730 | −907 | 47823 |
| Wages | 34806 | 15113 | | | |
| Profits | 22874 | 4948 | | | |
| 2000 total | 209730 | 47823 | | | 257553 |

(continued)

**Table A.4** (continued)

| | I | II | Final demand | Net export | Total |
|---|---|---|---|---|---|
| I | 177625 (13857) | 32686 (4882) | 35695 | 6703 | 252711 |
| II | | | 62820 | −2101 | 60719 |
| Wages | 39846 | 19104 | | | |
| Profits | 35239 | 8928 | | | |
| 2002 total | 252711 | 60719 | | | 313430 |
| I | 317851 (1966) | 41886 (28) | 70374 | 8645 | 438758 |
| II | | | 80160 | −2095 | 78064 |
| Wages | 55426 | 22805 | | | |
| Profits | 65479 | 13372 | | | |
| 2005 total | 438758 | 78064 | | | 516822 |
| I | 505437 (1806) | 52978 (3794) | 103924 | 22903 | 685243 |
| II | | | 101483 | 477 | 101960 |
| Wages | 80492 | 29555 | | | |
| Profits | 99314 | 19426 | | | |
| 2007 total | 685243 | 101960 | | | 787204 |

**Table A.5** Indices of structure and distribution

| | $\kappa_I$ | $\kappa_{II}$ | $\mu_I$ | $\mu_{II}$ | $\pi_I$ | $\pi_{II}$ | $\zeta$ | $\delta$ | $\sigma$ |
|---|---|---|---|---|---|---|---|---|---|
| 1981 | 2.988 | 1.744 | 1.060 | 0.770 | 0.266 | 0.281 | 2.488 | 0.174 | 0.463 |
| 1987 | 3.472 | 1.474 | 1.004 | 0.462 | 0.225 | 0.187 | 3.074 | 0.258 | 0.876 |
| 1990 | 3.789 | 1.690 | 0.956 | 0.488 | 0.200 | 0.181 | 3.505 | 0.209 | 0.808 |
| 1992 | 4.873 | 1.759 | 1.139 | 0.516 | 0.194 | 0.187 | 3.867 | 0.187 | 0.769 |
| 1995 | 4.433 | 2.099 | 0.955 | 0.624 | 0.176 | 0.201 | 4.176 | 0.196 | 0.855 |
| 1997 | 3.845 | 1.775 | 0.664 | 0.358 | 0.137 | 0.129 | 4.226 | 0.177 | 0.999 |
| 2000 | 4.368 | 1.837 | 0.657 | 0.327 | 0.122 | 0.115 | 4.386 | 0.140 | 0.907 |
| 2002 | 4.458 | 1.711 | 0.884 | 0.467 | 0.162 | 0.172 | 4.162 | 0.170 | 0.808 |
| 2005 | 5.735 | 1.837 | 1.181 | 0.586 | 0.175 | 0.207 | 5.620 | 0.196 | 0.892 |
| 2007 | 6.279 | 1.793 | 1.234 | 0.657 | 0.169 | 0.235 | 6.721 | 0.186 | 0.875 |

## A.4 The Turnpike for China's Economy

This section will investigate the turnpike for the 2-sector model of China's economy compiled as above.

Firstly, we will show how planning problems are formulated for simulation.

The actual chronological data in the simulation come from the data of years for which China's input-output tables were compiled (1987, 1990, 1992, 1995, 1997, 2000, 2002, 2005, and 2007). The objective function is such that maximises output of consumption goods in the final term, under the constraint of the system of difference inequalities, in which demand for the following period should not exceed supply of the present period.

### *A.4.1 Linear Programming Problem*

First, take an output vector $x(t)$ at time $t$. Since the demand of productive investment for the following period may not exceed the supply of the present period, the relationship between the following periods demand and that of the present period must be

$$x(t) \geqq Mx(t+1). \tag{A.4.1}$$

The initial state is already known to be $x(0)$, so the supply and demand relationship until time $n$ is as shown below.

$$\begin{aligned} x(0) &\geqq M_1 x(1), \\ x(1) &\geqq M_2 x(2), \\ &\vdots \\ x(n-1) &\geqq M_n x(n). \end{aligned} \tag{A.4.2}$$

These are systematised with matrices and vectors.

$$\begin{pmatrix} M_1 & & & \\ -I & M_2 & & \\ & \ddots & \ddots & \\ & & -I & M_n \end{pmatrix} \begin{pmatrix} x(1) \\ x(2) \\ \vdots \\ x(n) \end{pmatrix} \leqq \begin{pmatrix} x(0) \\ 0 \\ \vdots \\ 0 \end{pmatrix}. \tag{A.4.3}$$

Here, the objective function is taken that maximises the output of consumption goods $x_2(n)$ in the final year of the planning period. Again, within a constraint,

$$x(t+1) \geqq x(t), \ \ t = 0, 1, 2, \ldots, n-1. \tag{A.4.4}$$

That is, if the output in the target period is not lower than output in the previous period, the planning problem will boil down to

$$\max\{x_2(n) \mid Gx \leqq \boldsymbol{d}, x \geqq \boldsymbol{0}\}, \tag{A.4.5}$$

where

$$
G = \begin{pmatrix}
M_1 & & & \\
-I & M_2 & & \\
& \ddots & \ddots & \\
& & -I & M_n \\
-I & & & \\
I & -I & & \\
& \ddots & \ddots & \\
& & I & -I
\end{pmatrix}, \quad \boldsymbol{d} = \begin{pmatrix}
x(0) \\ 0 \\ \vdots \\ 0 \\ -x(0) \\ 0 \\ \vdots \\ 0
\end{pmatrix}.
$$

### *A.4.2 Preprocessing Data*

First, in order to solve the planning problem, the following preprocessing is carried out.

From Tables A.1 and A.4, $A^{**}$, $f^{**}$, and $L^{**}$ are computed. Then, $M$ can be formed. From coefficient $M$, von Neumann quantity ratio $q^c$ for each year can be computed.

If the annual data for coefficient $M^{**}$ is fixed as above, one can define $G$.

First, assuming an initial value of 1987, the $M_1, M_2, \ldots, M_n$ which are diagonally opposite to $G$ in (A.4.5) are the $M$ for 1988, 1989, ..., and 2007, respectively. $x(0)$ is the yield of capital and consumption goods in the 1987 2-sector table. Here, it is assumed that there was no technology innovation during the periods 1987–1989, 1990–1991, 1992–1994, 1995–1996, 1997–1999, 2000–2001, 2002–2004, and 2005–2006. In other words, it is assumed that

$$
\begin{gathered}
M_{87} = M_{88} = M_{89}, \;\; M_{90} = M_{91}, \;\; M_{92} = M_{93} = M_{94}, \;\; M_{95} = M_{96}, \\
M_{97} = M_{98} = M_{99}, \;\; M_{00} = M_{01}, \;\; M_{02} = M_{03} = M_{04}, \;\; M_{05} = M_{06}.
\end{gathered}
$$

Assuming that 2007 is the final year, this constitutes the complete set of data. $G$ becomes a matrix of $80 \times 40$, and $\boldsymbol{d}$ amounts to a column vector of $80 \times 1$. Thus, arrange these coefficients in the form (A.4.5), and the optimum values for each year can be computed. Taking into account the element of the price fluctuation (National Bureau of Statistics of China, China Statistical Yearbook), these are comparable with the values from the real economy (Table A.6).

**Table A.6** Computed von Neumann ratios

| 1981 | 1987 | 1990 | 1992 | 1995 | 1997 | 2000 | 2002 | 2005 | 2007 |
|---|---|---|---|---|---|---|---|---|---|
| 0.863 | 0.936 | 0.915 | 0.989 | 0.975 | 0.973 | 0.981 | 0.965 | 0.978 | 0.982 |
| 0.506 | 0.352 | 0.403 | 0.147 | 0.223 | 0.230 | 0.192 | 0.261 | 0.209 | 0.186 |

**Table A.7** Theoretical values $(x_1, x_2)$ and actual values $(x_1^*, x_2^*)$ (B.Y.)

| | 1987 | 1990 | 1992 | 1995 | 1997 | 2000 | 2002 | 2005 | 2007 |
|---|---|---|---|---|---|---|---|---|---|
| $x_1$ | 19364 | 31902 | 72023 | 129915 | 202524 | 367132 | 489370 | 561123 | 796354 |
| $x_2$ | 6299 | 13778 | 23077 | 29836 | 47854 | 67331 | 88126 | 96754 | 334367 |
| $x_1^*$ | 19364 | 32843 | 54396 | 126301 | 161601 | 209730 | 252711 | 438758 | 685243 |
| $x_2^*$ | 6299 | 9371 | 14068 | 30244 | 38243 | 47823 | 60719 | 78064 | 101960 |

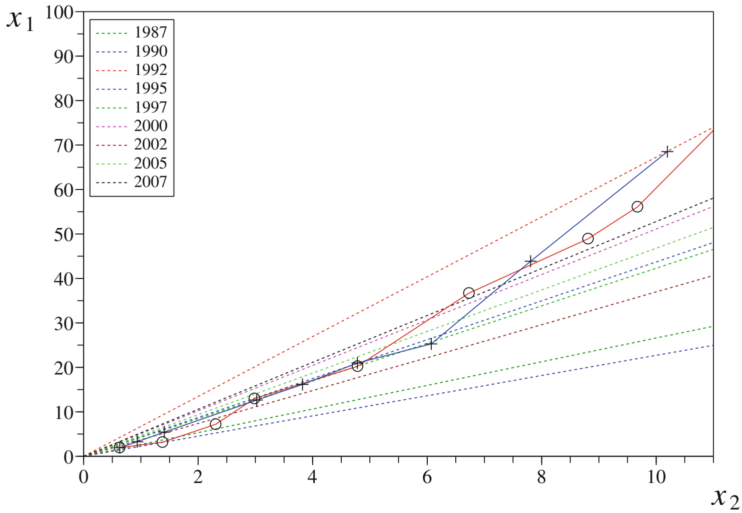


**Fig. A.11** China's von Neumann ratios and Turnpike paths (1987–2007)

The optimum values and actual values in China's economy are shown in Table A.7. The path of growth is shown in Fig. A.11.

## A.5 Concluding Remarks

This appendix expanded the discussion of Marxian 2-sector models, consumption-investment curves, the von Neumann ray and turnpike paths of China's economy.

First, in Sect. A.2 it became clear from the results of calculations using theoretical indices that (1) The accumulation rate in China's economy for the past few decades has been around 40%, and (2) From the early 1980s to the late 1990s, the maximum rate of profit tended to gradually decrease.

Next, it was established, based on the results of calculation using structural indices discussed in Sect. A.3, that (1) The relative importance of production goods is far greater than that of consumption goods. This supports the idea that China is still a developing country. (2) The rate of profit of both sectors tends to gradually decrease. (3) Until the latter half of the 1980s, the capital growth rate gradually decreased, and from the late 1990s till 2007, it tended to gradually increase again.

Finally, looking at turnpike paths in China's economy computed in Sect. A.4, one may say that the real economy has been close to the optimal path, a little overheated though.

In Fig. A.11, theoretical value: Red ($\circ$) line; Actual Value: Blue (+) line; von Neumann Ray: coloured dotted line.

# References

Abraham-Frois, G. and Berrebi, E. (1979), *Theory of Value, Prices and Accumulation—A Mathematical Integration of Marx, von Neumann and Sraffa*, Cambridge University Press.

Anderson, N., Saff, E. B. and Varga, R. S. (1979), "On the Eneström-Kakeya theorem and its sharpness," *Linear Algebra And Its Applications* 28, pp.5–16.

Asada, T. (1982), "Real Wage Rate, Rate of Profit and Rate of Exploitation in a Fixed Capital Economy," *Economic Studies Quarterly* 33(1), pp.55–66. (in Japanese)

Ben-Israel, A. and Greville, T. N. E. (2003), *Generalized Inverses. Theory and Applications*, 2nd ed. Springer.

Campbell, S. L. and Meyer, Jr. C. D., (1979), *Generalized Inverses of Linear Transformations,* Pitman.

Centre of Economic Forecasting, State Planning Commission of China and Department of Statistics on Balances of National Economy, State Statistical Bureau ed. (1987), *Input-Output Tables of China 1981*, University of Hawaii Press.

Chatelin, F. (1993), *Eigenvalues of Matrices*, John Wiley and Sons.

Department of National Accounts National Bureau of Statistics of China, *Input-Output Tables of China in 1987*, China Statistics Press.

Department of National Accounts National Bureau of Statistics of China, *Input-Output Tables of China in 1990*, China Statistics Press.

Department of National Accounts National Bureau of Statistics of China, *Input-Output Tables of China in 1992*, China Statistics Press.

Department of National Accounts National Bureau of Statistics of China, *Input-Output Tables of China in 1995*, China Statistics Press.

Department of National Accounts National Bureau of Statistics of China, *Input-Output Tables of China in 1997*, China Statistics Press.

Department of National Accounts National Bureau of Statistics of China, *Input-Output Tables of China in 2000*, China Statistics Press.

Fujimori, Y. (1982), *Modern Analysis of Value Theory*, Springer-Verlag.

Fujimori, Y. (1992a), "Building 2-Sector Schemes from the Input-Output Table: Computation of Japan's Economy 1960-1985," *Josai University Bulletion* 11(1), pp.1–12.

Fujimori, Y. (1992b), "Wage-Profit Curves in a von Neumann-Leontief Model: Theory and Computation of Japan's Economy 1970-1980," *Journal of Applied Input-Output Analysis* 1(1), pp.43–54.

Fujimori, Y. and Li, B. (2010), "Replacement of fixed capital as a Markov process," (in Chinese) *China Review of Political Economy* 1(4), pp.116–27.



B. Li, *Linear Theory of Fixed Capital and China's Economy*,
DOI 10.1007/978-981-10-4065-8

Fujimoto, T. (1975), "Duality and Uniqueness of Growth Equilibrium," *International Economic Review* 16(3), pp.781–91.

Gale, D. (1967), "On Optimal Development in a Multi-sector Economy," *Review of Economic Studies* 34(1), pp.1–18.

Gantmacher, F. R. (1959), *Matrix Theory* I, II, Chelsea.

Han, Z. and Schefold, B. (2006), "An empirical investigation of paradoxes: reswitching and reverse capital deepening in capital theory," *Cambridge Journal of Economics* 30, pp.737–65.

Hua, L. K. (1984), "On the mathematical theory of globally optimal planned economic systems," *Proc. Natl. Acad. Sci. USA* 81, pp.6549–53.

International Labour Office (2005), *Yearbook of Labour Statistics 2003* (*62th Issue*), The ILO Association of Japan.

Jorgenson, D. (1960), "A Dual Stability Theorem," *Econometrica* 28(4), pp.892–9.

Kalecki, M. (1954), *Theory of Economic Dynamics*, Allen & Unwin.

Kantorovich, L. V. [1959] (1965), *The Best Use of Economic Resources* (Translated by P. F. Knightsfield), Pergamon Press.

Koshimura, S. [1967] (1986), *Fixed Capital and Fluctuations* : *Theory of Crises and Waves*, Hakutou-Shoboh, Ltd.

Krüger, M. and Flaschel, P. (eds.), *Nobuo Okishio–Essays on Political Economy*: *Collected Papers*, Peter Lang.

Kurz, H. D. and Salvadori, N. (1995), *Theory of Production*: *A Long-Period Analysis*, Cambridge University Press.

Li, B. (2008), "China Input-Output Table and Linear Economic Theory," *Political Economy Quarterly* 45(2), pp.66–71. (In Japanese.)

Li, B. (2011a), "Economic durability of fixed capital," *Waseda Journal of Political Science and Economics* 381/382, pp.17–25.

Li, B. (2011b), "Estimation of Marginal Fixed Capital Coefficient and Wage-Profit Curves à la von Neumann-Leontief: A Case Study of China's Economy 1987-2000," *Waseda Economics Working Paper (WEWORP) Series*, No.11-001.

Li, B. (2012), *Linear Economic Theory and Turnpikes of China's Economy*: *In the Light of Marx, Sraffa and von Neumann*, Doctorate Dissertation, Graduate School of Economics, Waseda University. (in Japanese.)

Li, B. (2014a), Fixed Capital and Wage-Profit Curves a la von Neumann-Leontief: China's Economy 1987–2000, *Research in Political Economy* 29, pp.75–93.

Li, B. (2014b), Marx's Labour Theory of Value and Its Implications to Structural Problems of China's Economy, *Economic and Political Studies* 2(2), pp.139–50.

Li B. and Fujimori, Y. (2009), "A Multisector Linear Model with Fixed Capital," *Proceedings of Japan Society of Political Economy 57th Annual Conference* (II), pp.128–42.

Li, B. and Fujimori, Y. (2011), "Marx-Sraffa Equilibria and Eigenvalue Problems: A Moore-Penrose Pseudoinverse Approach," (in Japanese) *Political Economy Quaterly* 48(3), pp. 56–68.

Li, B. and Fujimori, Y. (2013), "Fixed Capital, Renewal Dynamics and Marx-Sraffa Equilibrium," in M. Kasamatsu ed., *Macro- and Micro Foundations of Economics*, pp.51–72, Waseda University Press.

Lü, Z. (2007), *Development and Application of the China Multi-area Multisector Dynamic Model in Consideration of the Gap between Areas*, (in Japanese), Graduate School of Frontier Sciences, University of Tokyo.

Mariolis, T. (2011), "A Simple Measure of Price-Labour Value Deviation," *Metroeconomica* 62(4), pp.605–11.

Mariolis, T. and Tsoulfidis, L. (2009),"Decomposing the Changes in Production Prices into 'Capital-Intensity' and 'Price' Effects: Theory and Evidence from the Chinese Economy," *Contributions to Political Economy* 28(1), pp.1–22.

Mariolis, T. and Tsoulfidis, L. (2010), "Measures of Production Price-Labour Value Deviation and Income Distribution in Actual Economies: A Note," *Metroeconomica* 61(4), pp.701–10.

Ministry of Finance of China (1992), *Information Concerning Promulgation of 'the Financial Institutions for Industrical Enterprises'*, [(92) CaiGongZi No.574].

Morishima, M. (1961), "Proof of a Turnpike Theorem: The No Joint Production Case," *Review of Economic Studies* 28(2), pp.89–97.

Morishima, M. (1964), *Equilibrium, Stability and Growth*: *A Multi-Sectoral Analysis*, Oxford University Press.

Morishima, M. (1973), *Marx's Economics*: *A Dual Theory of Value and Growth*, Cambridge University Press.

Nakatani, T. (1976), "The Amount of Labor and Price: In Case of Postwar Japan," *Economic Studies Quarterly* 27(1), pp.13–23. (in Japanese)

Nakatani, T. (1994), *Economics of Value, Price and Profit* (in Japanese), Keisou Shobou.

National Bureau of Statistics of China (ed.), *China Statistical Yearbook 2003*, China Statistics Press.

National Bureau of Statistics of China, Department of National Accounts (ed.), *Input-Output Tables of China in 2002*, China Statistics Press.

National Bureau of Statistics of China, Department of National Accounts (ed.), *Input-Output Tables of China in 2005*, China Statistics Press.

National Bureau of Statistics of China, Department of National Accounts (ed.), *Input-Output Tables of China in 2007*, China Statistics Press.

von Neumann, J. (1945/46[1937]), "A Model of General Economic Equilibrium," *Review of Economic Studies* 13(1), pp.1–9.

Nikaido, H. (1968), *Convex Structures and Economic Theory*, Academic Press.

Ochoa, E. (1989), "Values, Prices, and Wage-profit Curves in the US Economy," *Cambridge Journal of Economics* 13(3), pp.413–29.

Okishio, N. (1959), "Measurement of the Rate of Surplus Value," *Economic Review* 10(4), pp.297–303. (in Japanese)

Okishio, N. and Nakatani, T. (1975), "Profit and Surplus Labor: Considering the Existence of the Durable Equipments," (in Japanese) *Economic Studies Quarterly* 26(2), pp.90–6.

Okishio, N. and Nakatani, T. (1993), "A Measurement of the Rate of Surplus Value," in Krüger, M. and P. Flaschel (eds.), *Nobuo Okishio–Essays on Political Economy*: *Collected Papers*, pp.61–73, Peter Lang.

Ortega, J. M. and Rheinboldt, W. C. (2000), *Iterative Solution of Nonlinear Equations in Several Variables*, SIAM.

Parys, W. (1982), "The Deviation of Prices from Labor Values," *American Economic Review* 72(5), pp.1208–12.

Pasinetti, L. L. (1977), *Lectures on Theory of Production*, Macmillan.

Paton, W. A. and Littleton, A. C. (1940), *An Introduction to Corporate Accounting Standards*, American Accounting Association.

Qi, S. (2003), "A Brief Introduction to the Formation and Application of Input-Output Table in China,"(in Chinese) *China Statistics* 5, pp.21–2.

Rao, C. R. and Mitra, S. K. (1971), *Generalized Inverse of Matrices and its Applications*, John Wiley and Sons.

Schefold, B. (1980), "von Neumann and Sraffa: Mathematical Equivalence and Conceptual Difference," *Economic Journal* 90, pp.140–56.

Schefold, B. (1989), *Mr Sraffa on Joint Production and Other Essays*, Unwin Hyman.

Schefold, B. (1997), *Normal Prices, Technical Change and Accumulation*, St Martins Press.

Sheskin, D. J. (2000), *Handbook of Parametric and Nonparametric Statistical Procedures* (*Second Edition*), Chapman & Hall/Crc.

Seneta, E. (1980), *Non-Negative Matrices and Markov Chains*, 2nd ed., Springer.

Sraffa, P. (1960), *Production of Commodities by Means of Commodities*: *Prelude to a Critique of Economic Theory*, Cambridge University Press.

State Council of China (2007), *Regulations on the Implementation of Enterprise Income Tax Law of the People's Republic of China*, [Decree No.512].

Steindl, J. (1952), *Maturity and Stagnation in American Capitalism*, Blackwell.

Strang, G. (1976), *Linear Algebra and Its Applications,* Academic Press.

Traub, J. F. (1964), *Iterative Methods for the Solution of Equations*, Prentice Hall, Englewood Cliffs.

Tsoulfidis, L. (2008), "Price-value Deviations: Further Evidence from Input-Output Data of Japan," *International Review of Applied Economics* 22(6), pp.707–24.

Tsoulfidis, T. and Rieu, D. -M. (2006), "Labor Values, Prices of Production, and Wage-Profit Rate Frontiers of the Korean Economy," *Seoul Journal of Economics* 19(3), pp.275–95.

Tsoufidis, L. and Mariolis, T. (2007), "Labour Values, Prices of Production and the Effects of Income Distribution: Evidence from the Greek Economy," *Economic Systems Research* 19(4), pp.425–37.

Tsukui, J. (1967), "The Consumption and the Output Turnpike Theorems in a von Neumann Type Model: A Finite Term Problem," *Review of Economic Studies* 34(1), pp.85–93.

Wolff, E. N. (1979), "The Rate of Surplus Value, the Organic Composion, and the General Rate of Profit in the U.S. Economy, 1947-67," *American Economic Review* 69(3), pp.329–41.

Yamada, K. and Yamada K. (1961), "Extended reproduction and replacement of fixed capital: an application of Kakeya's theorem to difference equation," (in Japanese), *Hitotsubashi Review* 46(5), pp.531–8.